$2b$ or Not $2b$: Supporting Learners Who Struggle in Mathematics

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M4 Conference

Overview of Session

This session focuses on ways to support students who struggle in Tiers 1 and 2 to become:

- Confident in mathematics
- Successful with <u>rigorous</u> mathematics
- Capable of working with complex mathematical ideas

Reflect

- When you think of learners who struggle, what types of instruction and content do you feel are important for them?

Making Cents

1. Take out some coins
2. Multiply the value of the coins in cents by 4
3. Add 10 to the product
4. Multiply your answer by 25
5. Add 115 to your answer
6. Add your age in years
7. Subtract the number of days in a normal year

Making Cents Debrief

- What do you notice about your answers?
- How can you describe what you notice to someone who is not present?

How do you describe what you see in terms of student learning?

Types of Understandings

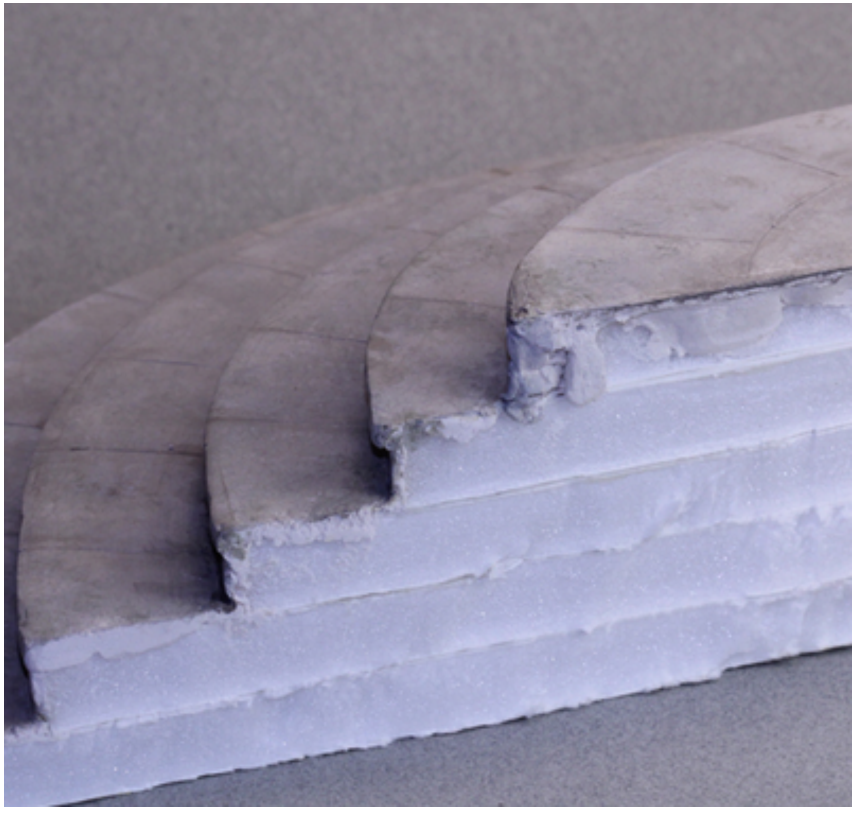
Procedural - Student can perform a computation or algorithm by following a series of prescribed steps

Conceptual - Student understands the basis of why a computation or algorithm works. They can apply it later without reteaching. Student can identify, describe, and explain a big idea related to a topic or a class of problems

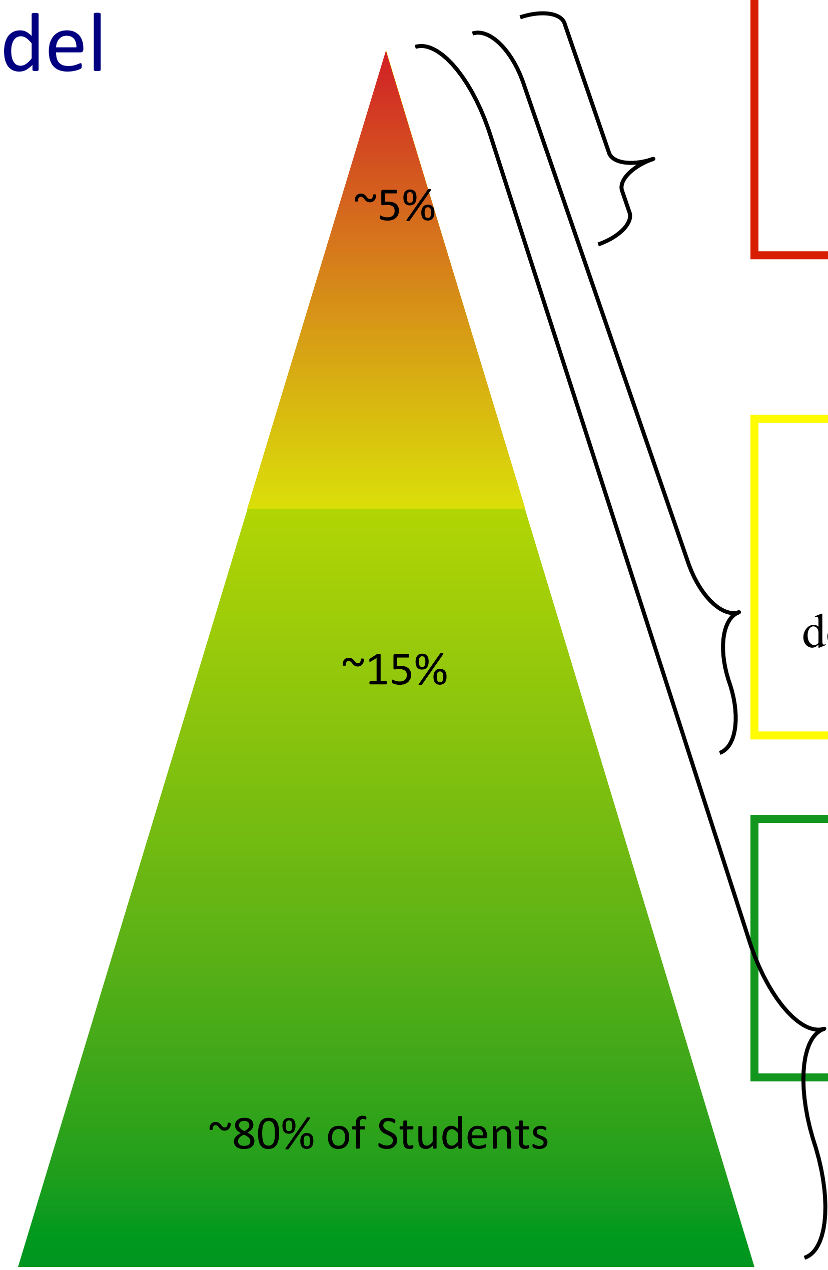
Problem solving - Student can solve a problem when there is no specific solution pathway or algorithm

Tiered Support Systems

- What types of tiered support do you offer in your school or district?



3-Tiered Support Model



<u>Tertiary Prevention</u>:
Specialized & individualized strategies for students who don't respond to Tier 2 supplementary support - INTENSIVE

<u>Secondary Prevention</u>:
Supplementary strategies for students who do not respond to core instruction - TARGETED

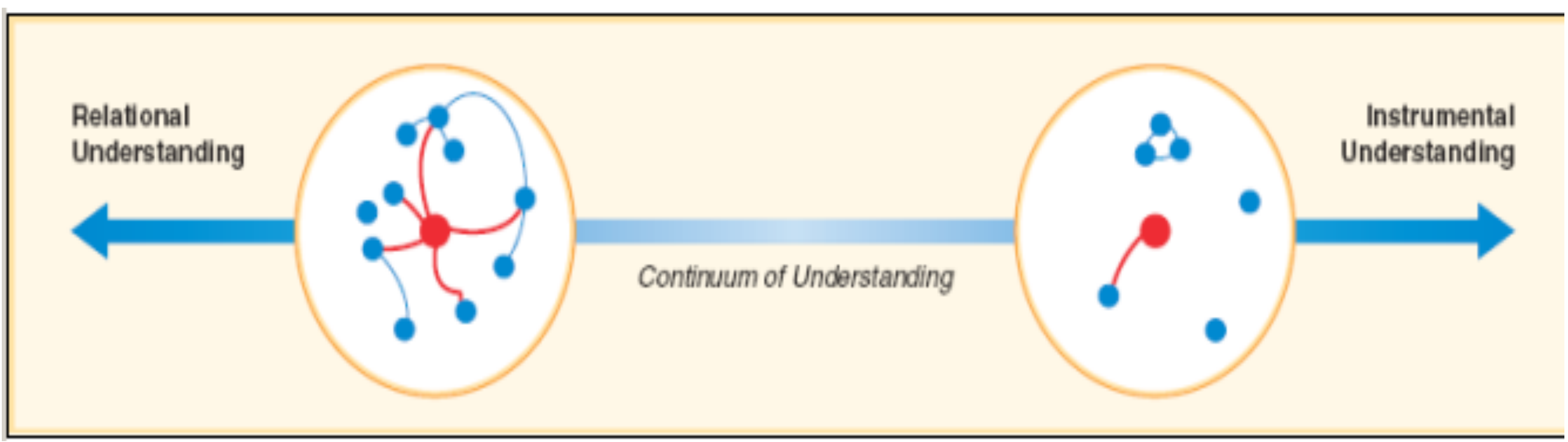
<u>Primary Prevention</u>:
High quality engaging core instruction - UNIVERSAL

Models within Tier 2

- Problem-solving model: School-based team uses multiple sources of data to determine interventions, often individualized
- Functional assessment model: Students assessed before intervention, at intervals throughout intervention (behavior focused)
- Standard protocol model: Intervention is a research-based, well-formulated curriculum

What are characteristics of students in Tier 2?

Compared to students who are not struggling, their brains might look very different!



Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

- What the problem is not—
  - difficulty reading, paying attention, or following directions
- What the problem is—
  - **Underdeveloped cognitive structures** (the mental processes necessary to connect new information with prior knowledge)

Geary, David C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37(*1), 4-15.

Components of A Strong Multi-Tiered System of Support Model

Uses a co-teaching approach as a collaboration between general education and special education

Includes research based teaching practices

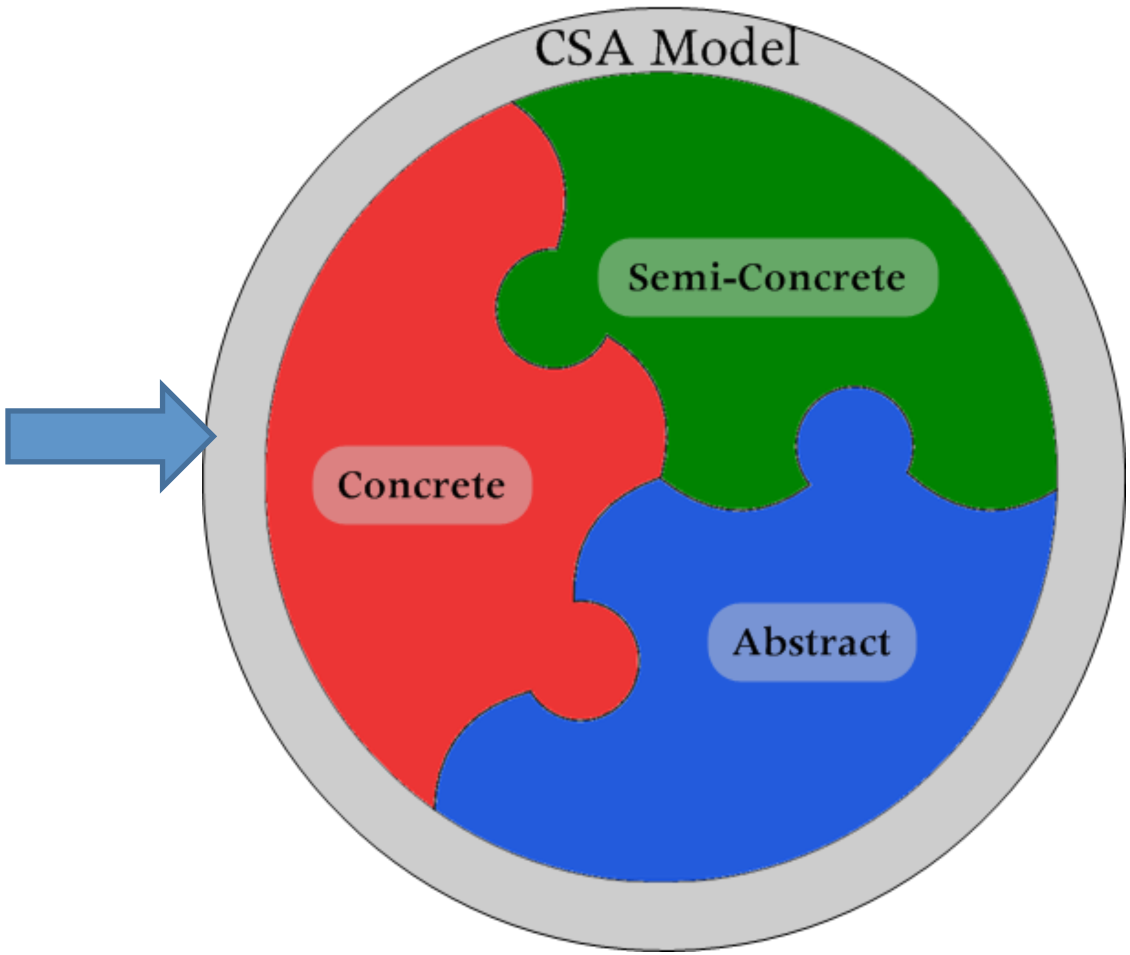
Uses screening and progress monitoring to Instruct with a preventative approach

Builds from students' strengths

Uses diagnostic assessments to align intervention

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies:Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.

CSA: Concrete—Semi-Concrete—Abstract



Intervention Recommendations from Research

- **C**oncrete—**S**emi-concrete—**A**bstract (CSA) representations
- Explicit instruction (not direct instruction)
- Underlying mathematical structures
- Examples (and counterexamples)
- Feedback – Not teacher to student but students' feedback to other students and teacher on what they know and don't know

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.
Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* New York: Routledge.

Create Mental Residues

- Establishes foundational understanding
- Models the physical action is the important
- Does not fade away or disappear
- Supports their thinking about the operation

Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput,
J. J., Carraher, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahweh, NJ: Erlbaum.

Characteristics of learning

- Introduce every topic with problem solving
- Every lesson includes five forms of communication
  - Reading
  - Speaking
  - Critical listening
  - Writing
  - Multiple representations
- Topics are connected
- Students have 8–15 days to move a concept to a skill
- Challenging problems for all students

Which comes first:

Concepts or skills?

What happens with procedural teaching

Focus on Skills

$$\frac{1}{12} + \frac{7}{8}$$

Focus on Skill

$$\frac{1}{12} + \frac{7}{8} = \frac{23}{24}$$

Focus on Sense Making

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. $\frac{1}{2}$
D. 1

The Two Worlds Collide: Sense Making Meets Skill

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. ½
D. 1

Explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

Petit, Laird, & Marsden, 2010

Mathematical Outcomes

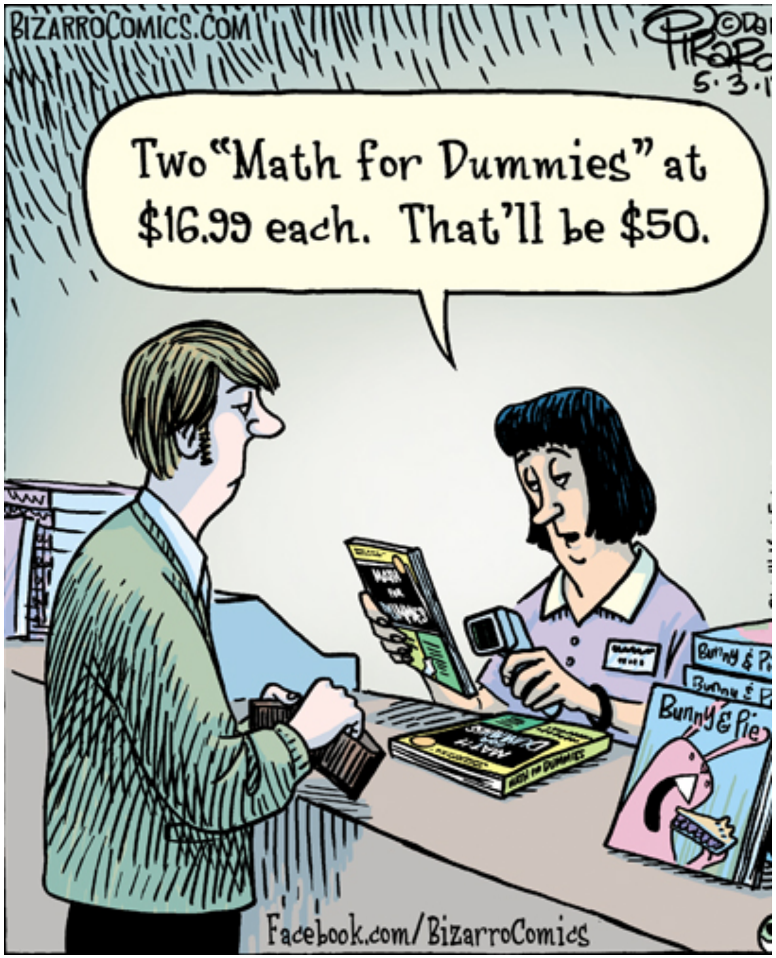
Small, fragmented, and isolated skills are not the desired outcome for students who struggle with math.

Quote from A Learner who Struggled

"As an adult, when I look back at math, I hated it. It was a mystery. Everything seemed to be hocus-pocus math. Do this, do that, and presto – you get the right answer. But I had too much stuff to memorize. I couldn't keep track of it. It was like being in a forest and seeing only the leaves on the trees and forgetting there's a forest!"

Personal communication, Mr. Burns (pseudonym)

Mathematical Outcomes



Goals for students who struggle:

- See patterns and generalize those patterns
- Apply the generalizations and patterns to other problems (see connections)
- Use reasoning and intuition when possible

Rules

- With the person sitting next to or around you, decide if the rules shown are always true.
- If it is not always true, find a counterexample.

- Addition and multiplication make larger.
- When you multiply by 10, add 0 to the end of the number.
- Two negatives make a positive.
- The longer the number, the larger the number.

Addition and multiplication make larger.

$32 + 67 = 99$

$15 \times 10 = 150$

$-3 + (-14) = -17$

$$\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$$

When you multiply by 10, add a 0 at the end of the number.

15 x 10 = 150

4.5 x 10 = 45.0

Two negatives make a positive.

$$-8 \times (-3) = 24$$

$$-8 + (-3) = -11$$

The longer the number, the larger the number.

1,278,931 > 1,469

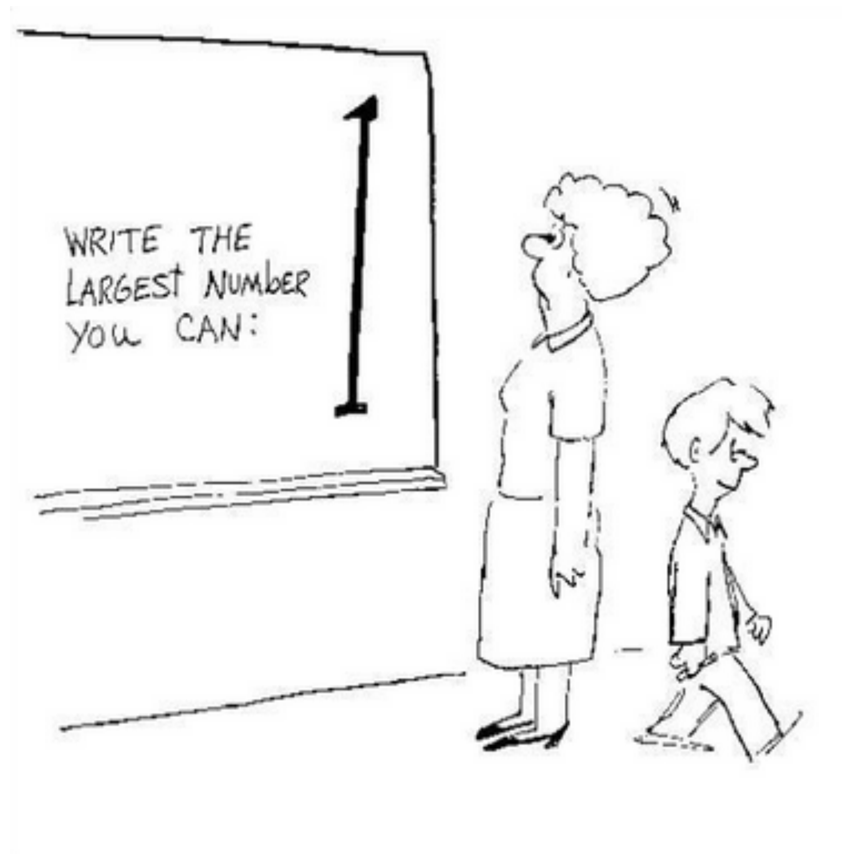
1.3 > 1.0118743

So---what do we do??

- Focus on BIG Ideas and generalizations

Beginning number ideas

- Counting
  - Rote
  - Cardinality
  - Ordinality



Sammi-Jo said, "I counted 4 things." "No," said Henna. "There are 8 things."

Is it possible that Sammi-Jo and Henna counted the same group of objects? why or why not?

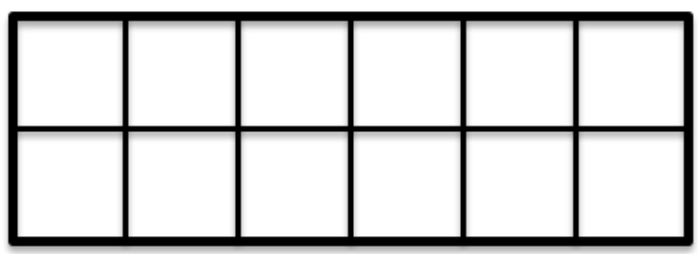
Dougherty, B. J., Flores, A., Sophian, C., & Everett, L. (2010). *Essential understandings: Engaging teachers with mathematics, PreK–2 number concepts*. Reston, VA: National Council of Teachers of Mathematics.

Counting Task

1. How many ways can you find to measure the area in the rectangle below?

2. What would the units look like?

3. How would the units compare?



Dougherty, Zenigami, & Okazaki (2007). *Measure Up*, grade 1. University of Hawaii.

Units are important!

- Instruction often does not explicitly provide opportunities to discuss the importance of unit.

- Unit is a fundamental concept in mathematics.

What do we assume?

$3 < 8$

What first graders had to say. . .

$3 < 8$

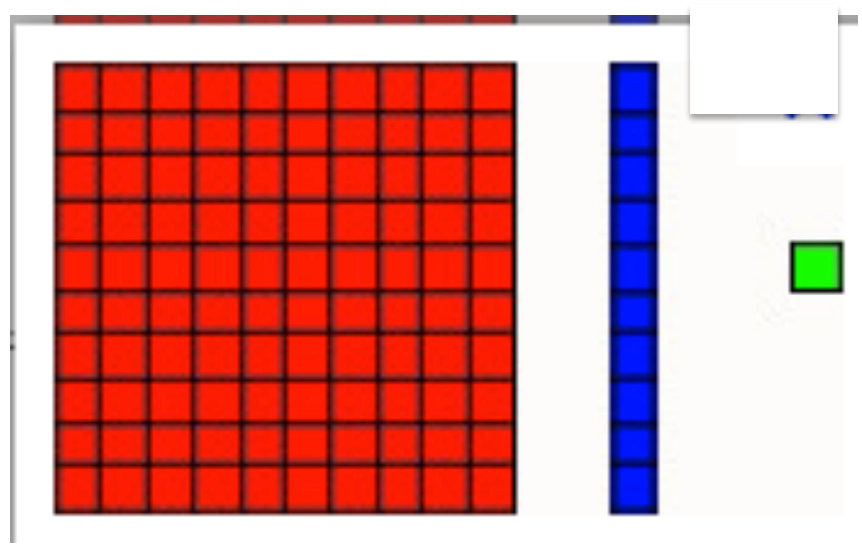
Richard: You can’t really tell. ‘Cause you could have 3 really really really big units and 8 really really really small units. Then 8 would be less than 3.

Reed: Yeah, but if it’s on a number line then it’s true ‘cause all the units are the same size.

Whole Numbers

If you wanted to use base ten blocks, how could you model 125?

Work with a partner to think about how many different ways you could model 125.



Whole Numbers

- Decomposition
  - Writing equations to symbolize the representations

$$125 = 100 + 10 + 10 + 5$$

$$125 = 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1$$

$$125 = 60 + 60 + 5$$

Comparing Whole Numbers

- What can you say about the relationship between 125 and 152?
- What statements could you write or say?

Comparing Whole Numbers

- What can you say about the relationship between 125 and 152?
- What statements could you write or say?

$125 \neq 152$

$152 \neq 125$

$125 < 152$

$152 > 125$

Comparing Whole Numbers

$125 \neq 152$

$152 \neq 125$

$125 < 152$

$152 > 125$

$125 < 152$ by 27

$152 > 125$ by 27

Comparing Whole Numbers

$125 \neq 152$

$152 \neq 125$

$125 < 152$

$152 > 125$

$125 < 152$ by 27

$152 > 125$ by 27

$152 = 125 + 27$

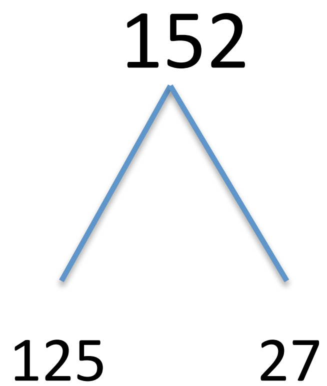
$152 = 27 + 125$

$152 - 27 = 125$

$152 - 125 = 27$

Part-Whole Diagrams

125 < 152 by 27



Part-Part-Whole Problems

Lynnette has 14 fiction and 23 nonfiction books. How many books does she have?

Lynnette and her friend Victoria put 37 books into a backpack. Lynnette put in 14 books. How many books did Victoria put in the backpack?

If there are 37 books in the backpack. What are the different combinations that both girls could have placed in the backpack?

Equal Sign

Are students acquiring an appropriate understanding of the equal sign when you ask them to explain their thinking?

Are they comfortable using operations on both sides of the equal sign and can use the meaning of equal as “is the same as?”

Equal Sign–Two Levels of Understanding

Operational: Students see the equal sign as signaling something they must "do" with the numbers such as "give me the answer."

Relational: Students see the equal sign as indicating two quantities are equivalent, they represent the same amount. More advanced relational thinking will lead to students generalizing rather than actually computing the individual amounts. They see the equal sign as relating to "greater than," "less than," and "not equal to."

Knuth, E. et. al (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, *13*, 514–519.

Why is understanding the equal sign important?

Table 1 Percent of students at each grade level who provided each type of equal sign definition as their best definition ($n = 375$)

Best Definition	Grade 6	Grade 7	Grade 8
Relational	29	36	46
Operational	58	52	45
Other	7	9	8
No response/ don't know	6	3	1

Knuth, E. et. al (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, *13*, 514–519.

Which number sentences would students say are True? False?

27 = 27

22 + 5 = 4 + 23

25 + 1 = 27

27 = 22 + 5

- Why?
- What would confuse them?

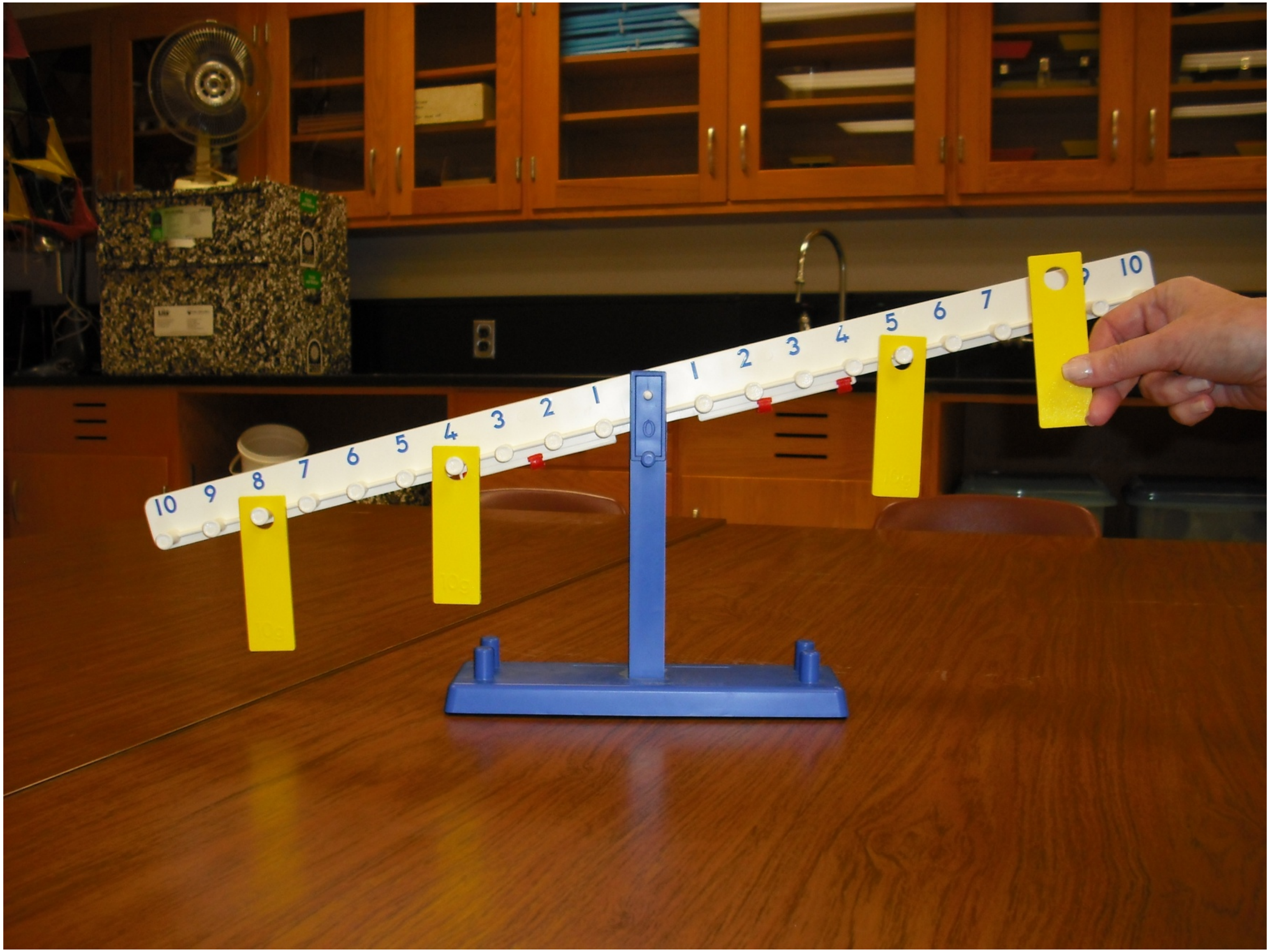
How is the equal sign interpreted?

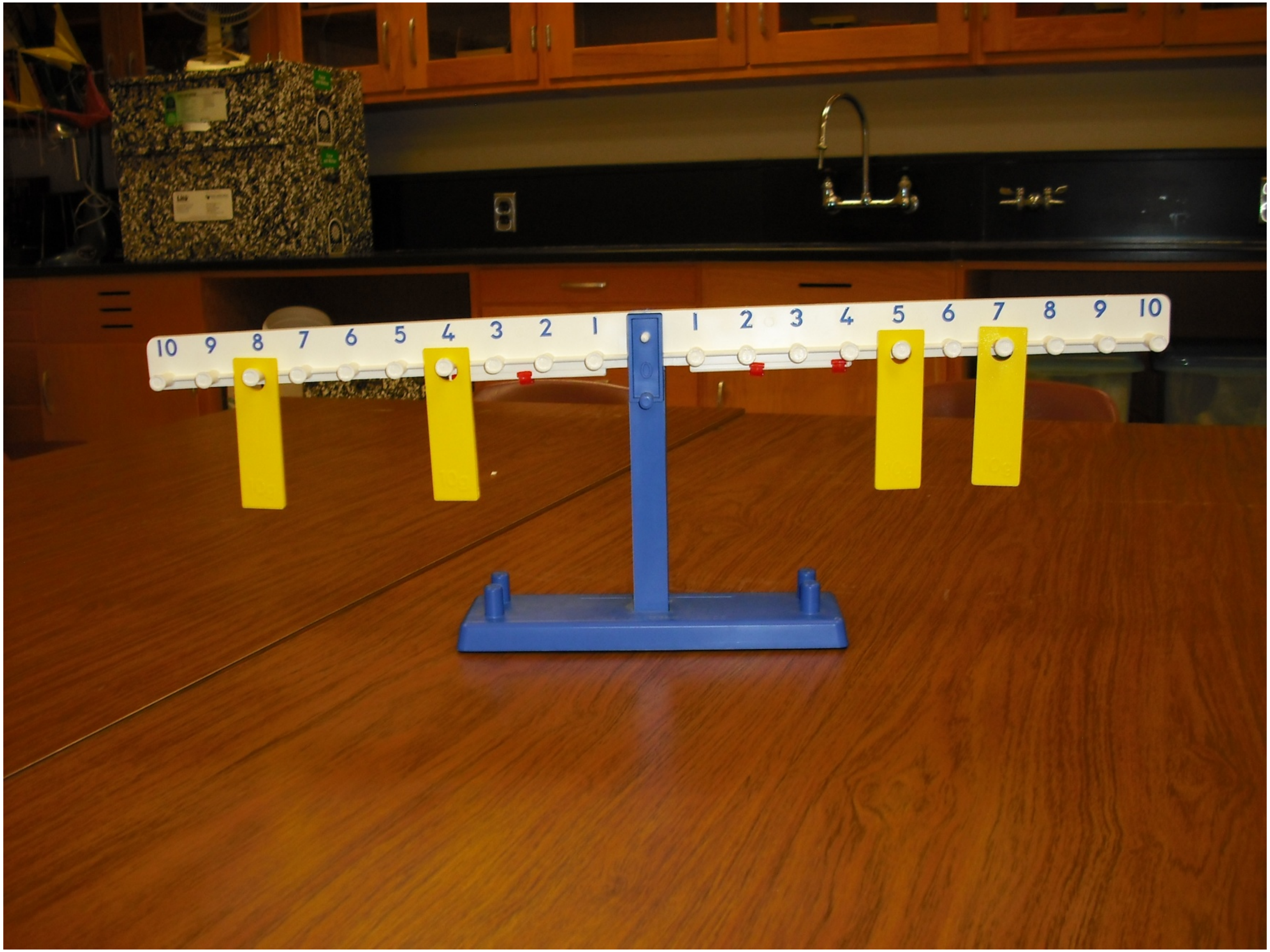
Given the task:

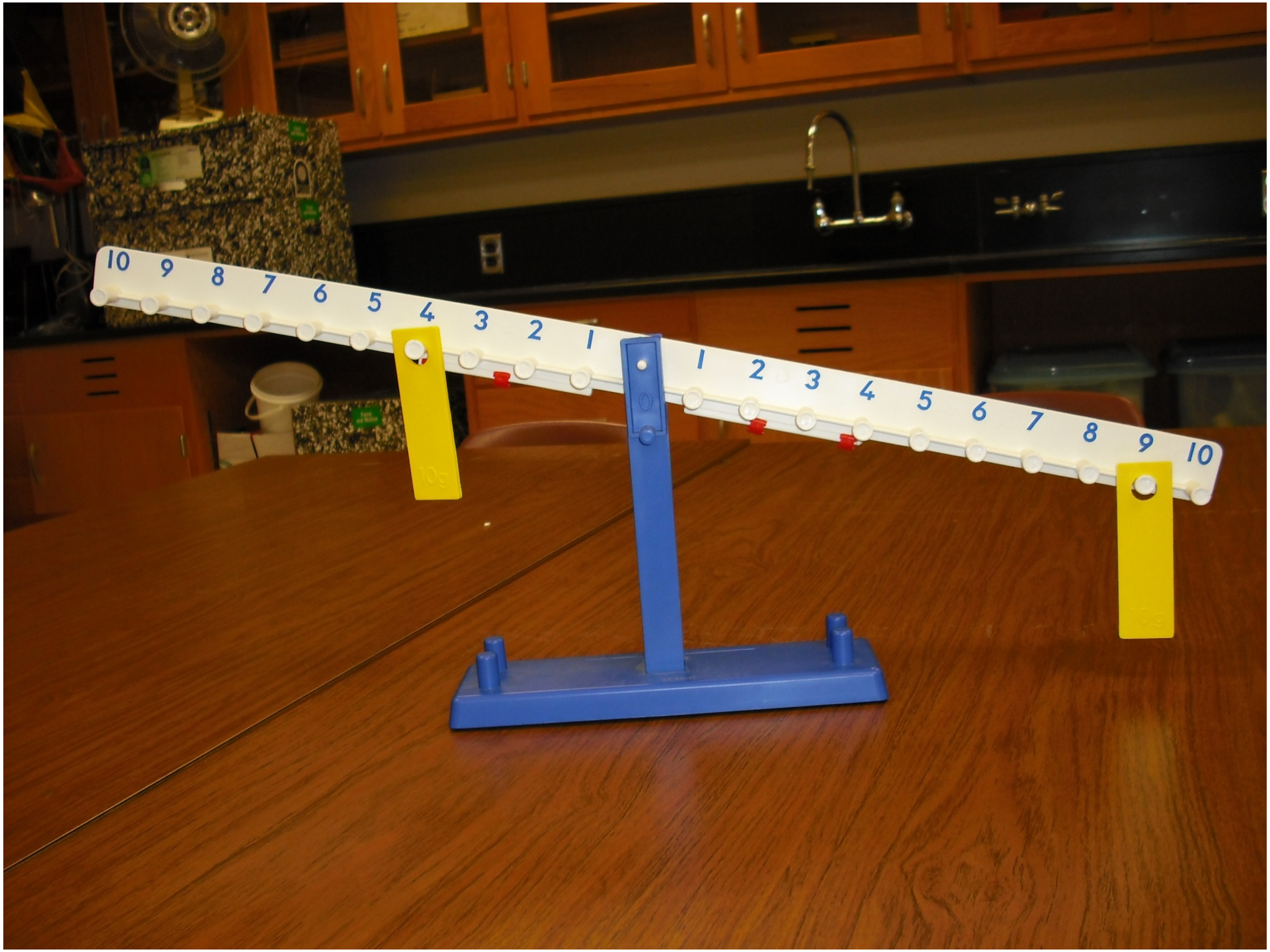
$$8 + 4 = \square + 5$$

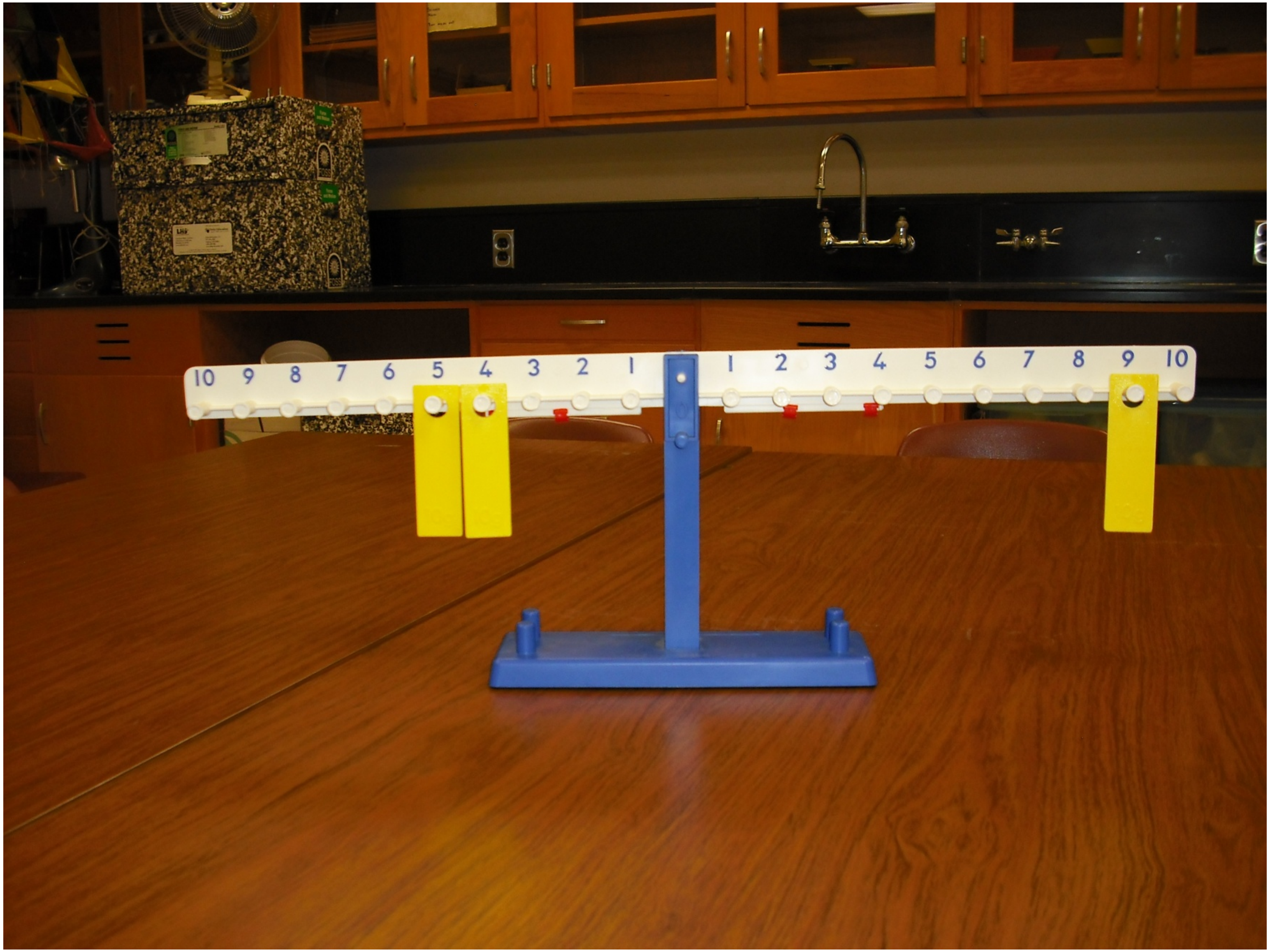
How do you think students will respond?

$$8 + 4 = \boxed{12} + 5$$


10 9 8 7 6 5 4 3 2 1
0
1 2 3 4 5 6 7 10


10 9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10


10 9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10
10g


10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10

Problem Solving: Application/Routine Problems

At all grades, students who struggle see each problem as a separate endeavor.

They focus on steps to follow rather than the behavior of the operations associated with the problem.

They tend to use guess and test – (disconnected thinking).

They need to focus on actions, representations, and general properties of the operations.

What is the action?

Louise had 11 baseball cards. Elliott gave her 6 more. How many does she have now?

Louise had 11 baseball cards. Elliott gave her some more. How many did Elliott give her?

Louise had some baseball cards. Elliott gave her 6 cards. Now she has 17. How many did she strt with?

What is the action?

Louise had 11 baseball cards. Elliott gave her 6 more. How many does she have now?

$$11 + 6 = ?$$

Louise had 11 baseball cards. Elliott gave her some more. She has 17 now. How many did Elliott give her?

$$11 + \square = 17$$

Louise had some baseball cards. Elliott gave her 6 cards. Now she has 17. How many did she start with?

$$\square + 6 = 17$$

What models of subtraction do these problems represent?

There were 17 boys playing tag. Twelve boys went home. How many boys are still playing tag?

Tom has 5 brothers. Juan has 3 brothers. How many more brothers does Tom have than Juan?

Jesse has 14 golf balls. Teva has 23 golf balls. How many more golf balls does Jesse need to have as many as Teva?

What models of subtraction do these problems represent?

There were 17 boys playing tag. Twelve boys went home. How many boys are still playing tag?

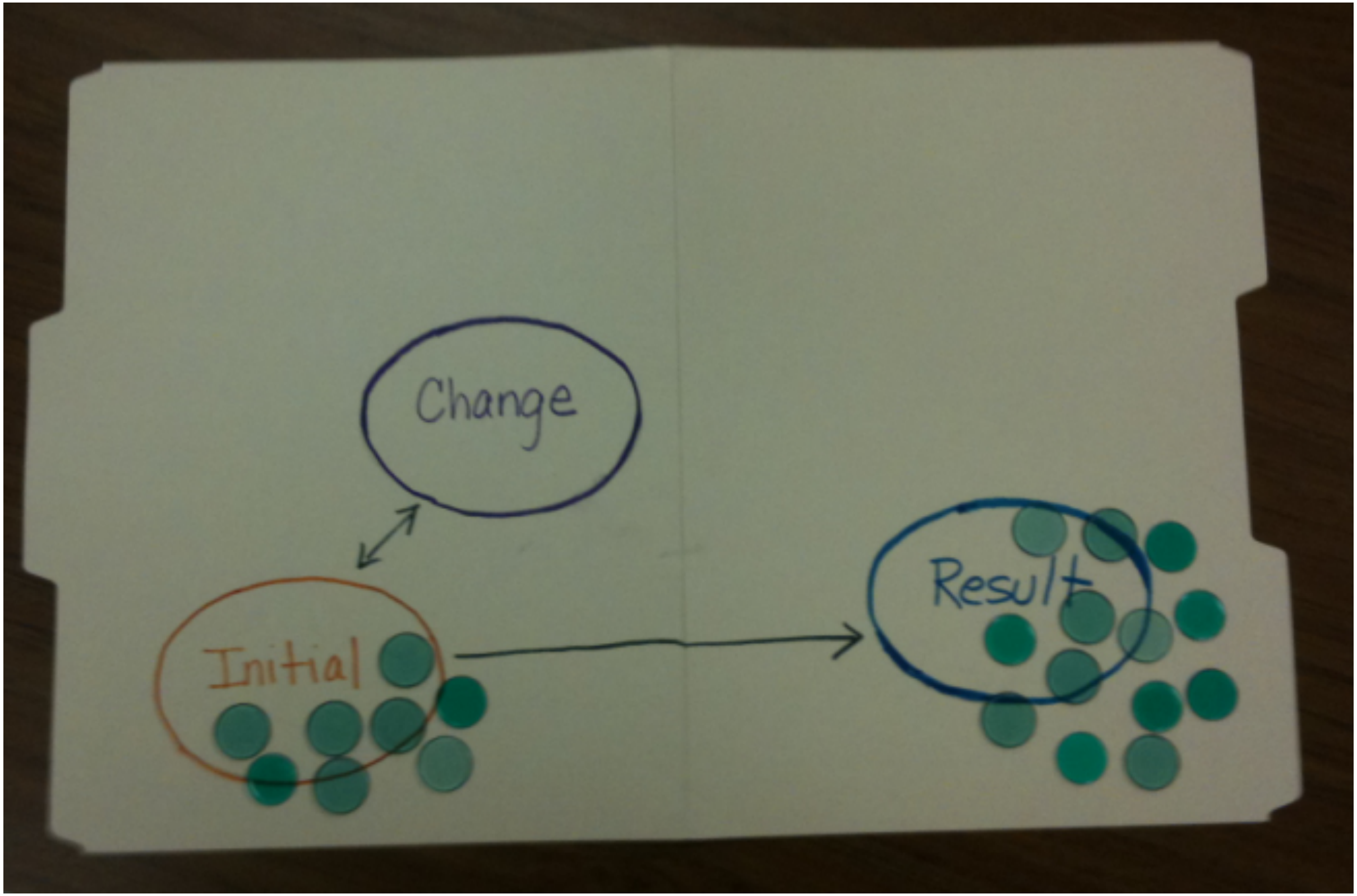
$$17 - 12 = 5$$

Tom has 5 brothers. Juan has 3 brothers. How many more brothers does Tom have than Juan?

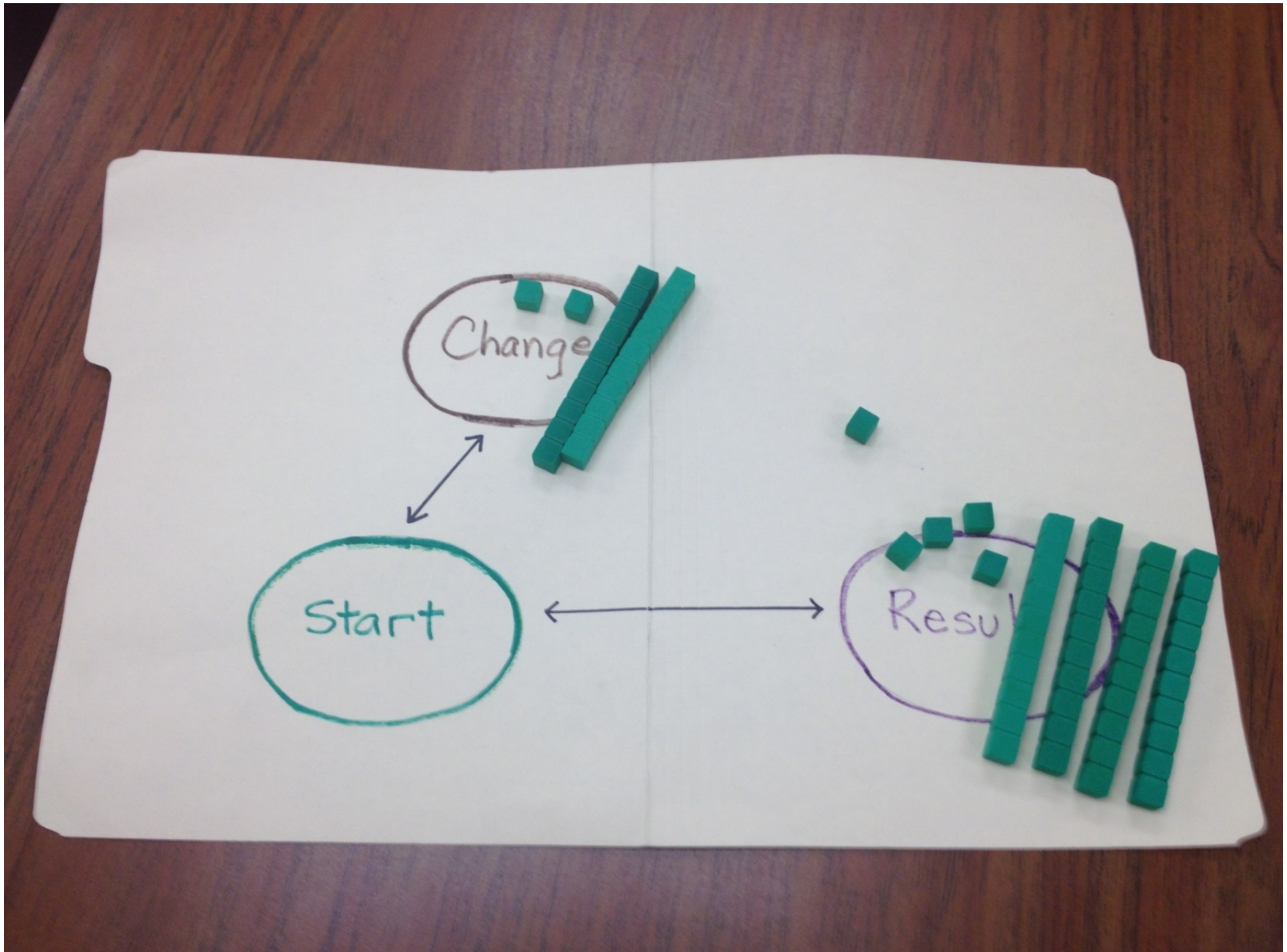
$$3 + \square = 5$$

Jesse has 14 golf balls. Teva has 23 golf balls. How many more golf balls does Jesse need to have as many as Teva?

$$14 + \square = 23$$


Change
Initial
Result

The Graphic Organizer that Keeps on Going!!!



Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$

Common Multiplication and Division Situations

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Find-A-Place

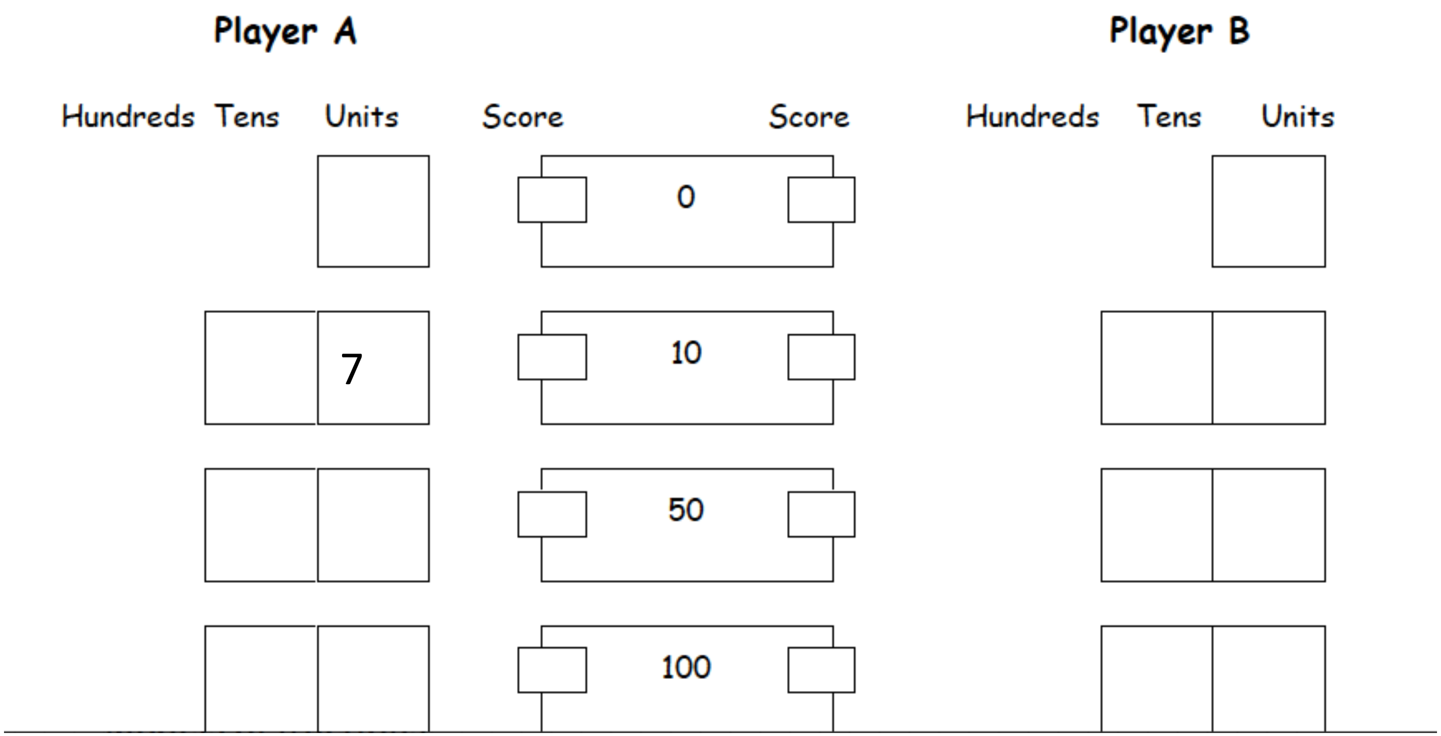
- You will need to work in pairs.

Find-A-Place

FIND A PLACE

(2 Players)

Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).



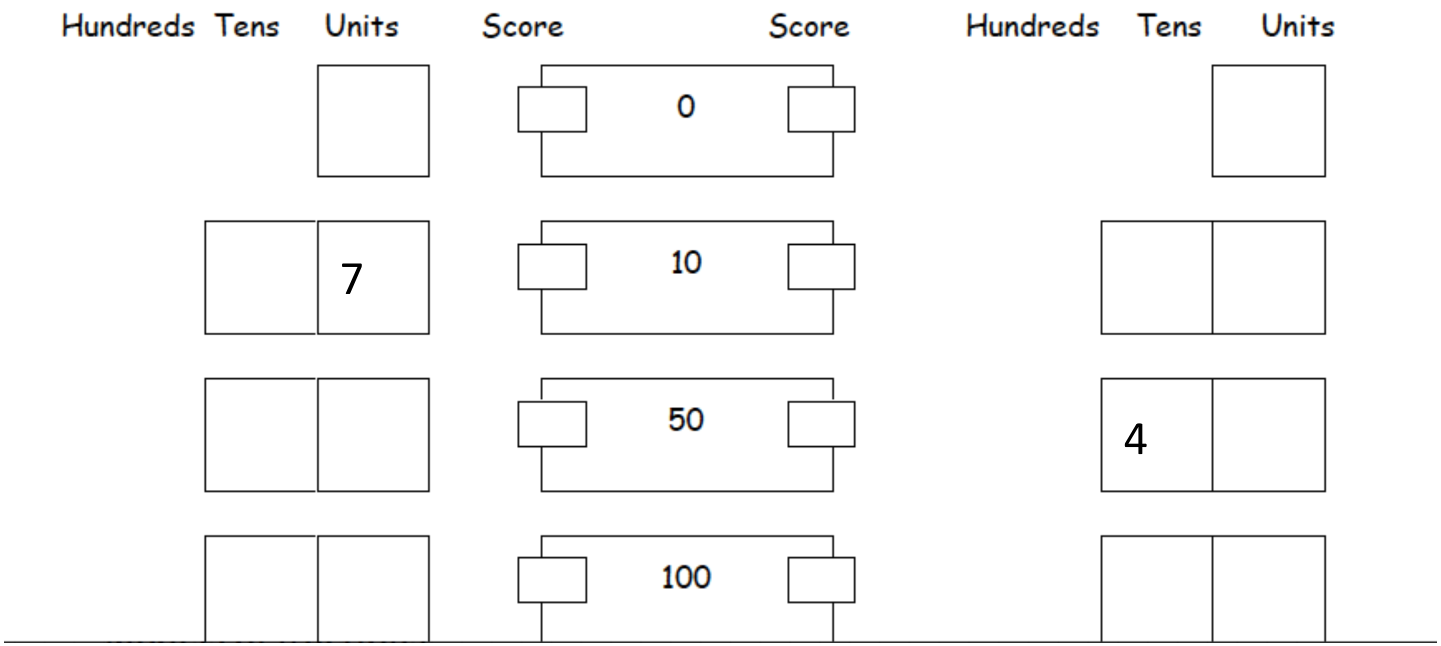
Find-A-Place

FIND A PLACE

(2 Players)

Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).

Player A **Player B**



Find-A-Place

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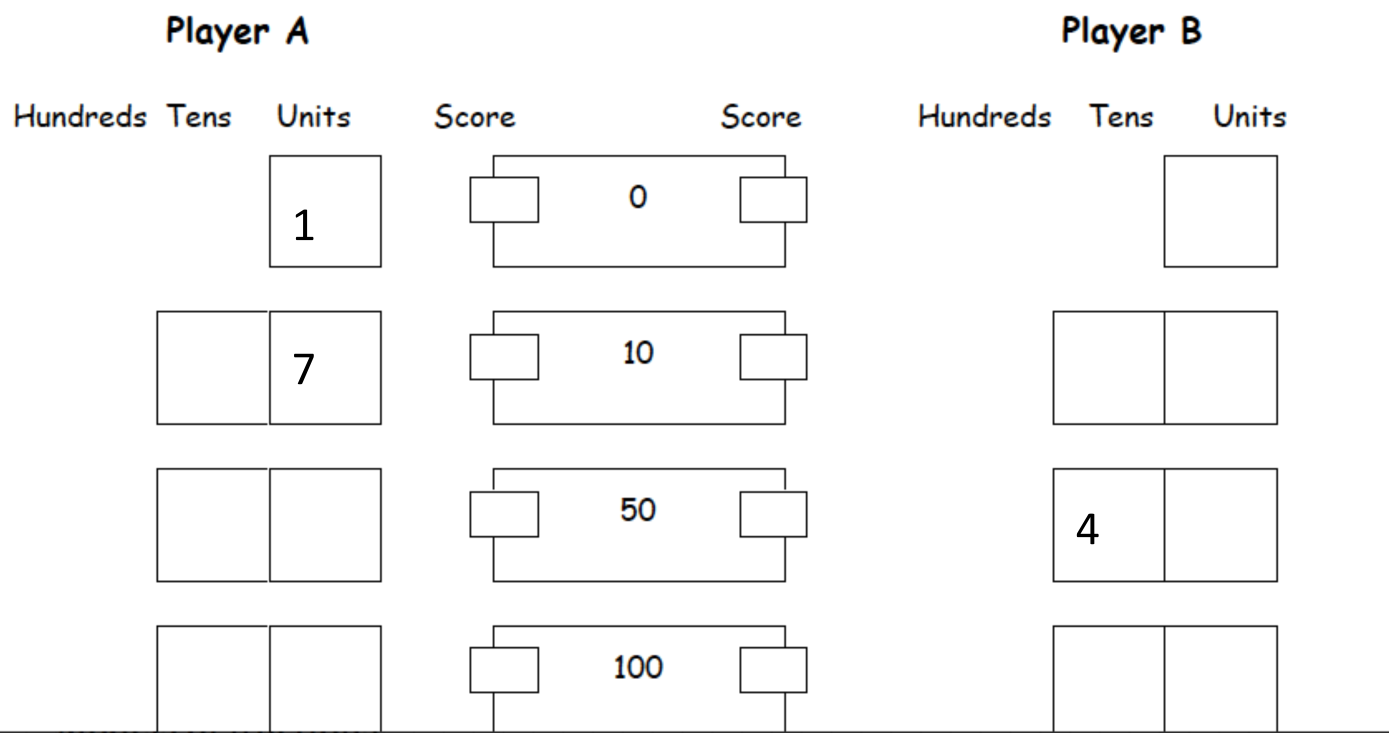


Find-A-Place

FIND A PLACE

(2 Players)

Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).

Player A						Player B		
Hundreds	Tens	Units	Score		Score	Hundreds	Tens	Units
		1		0				
		7		10				
				50			4	
				100			8	

Find-A-Place

FIND A PLACE

(2 Players)

Use 40 cards numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (four of each).

Player A Hundreds	Player A Tens	Player A Units	Score		Score	Player B Hundreds	Player B Tens	Player B Units
		1		0				0
	1	7		10			0	2
	5	5		50			4	8
	7	8		100			8	1

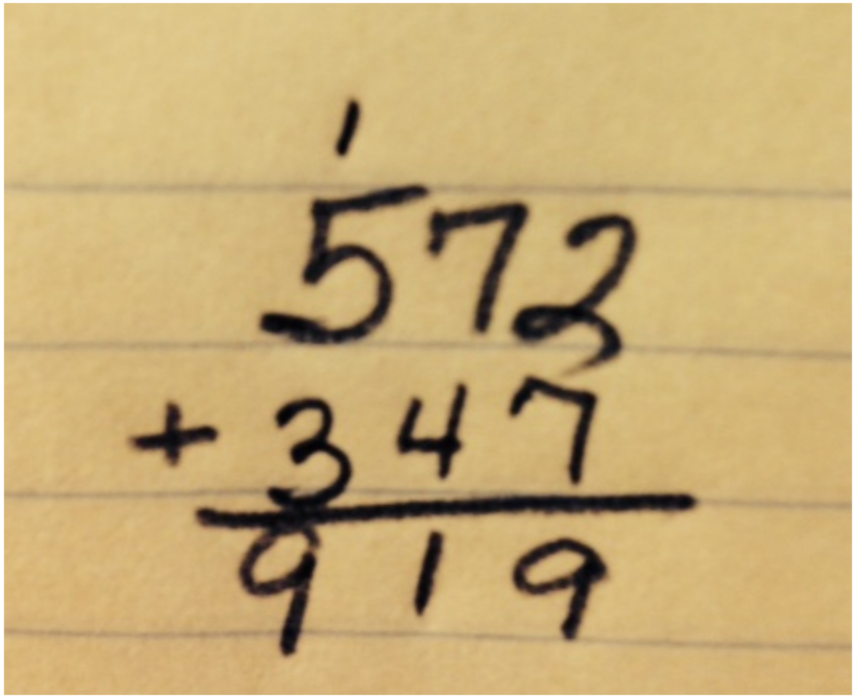
Number Development

Solve in at least three ways:

$$\begin{array}{r} 572 \\ +\ 347 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 862 \\ -\ \ 295 \\ \hline \end{array}$$

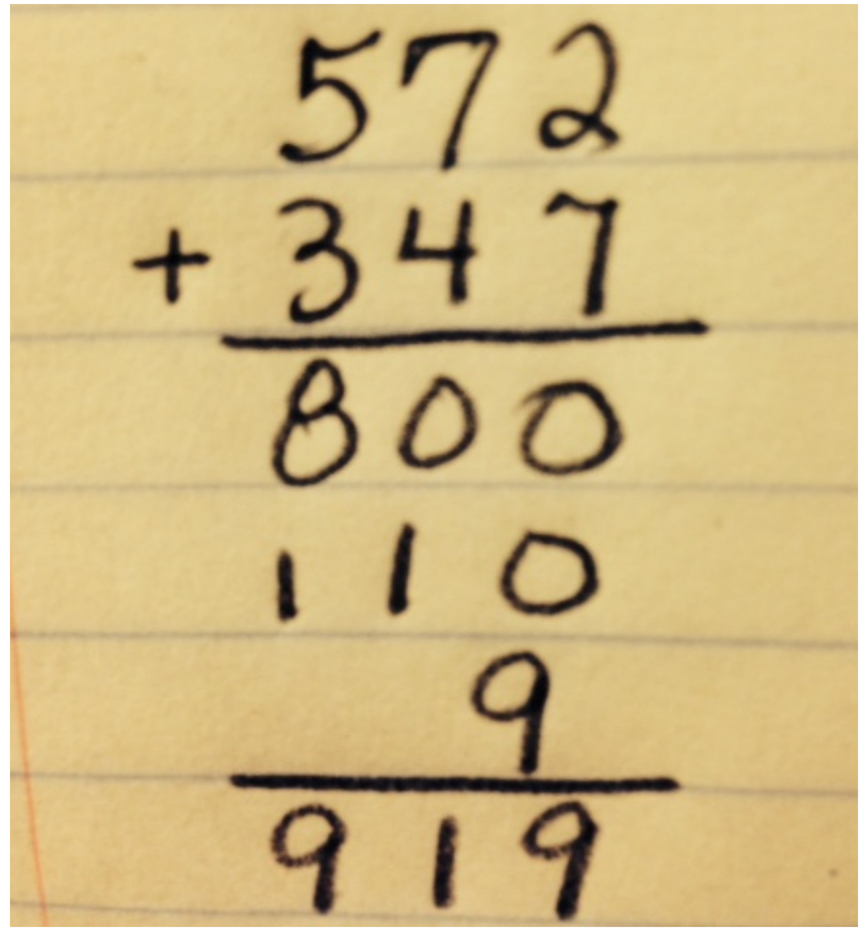
Algorithms

- Traditional
  - Does not preserve placement
  - Does not use number sense or intuition
  - Directionality issues



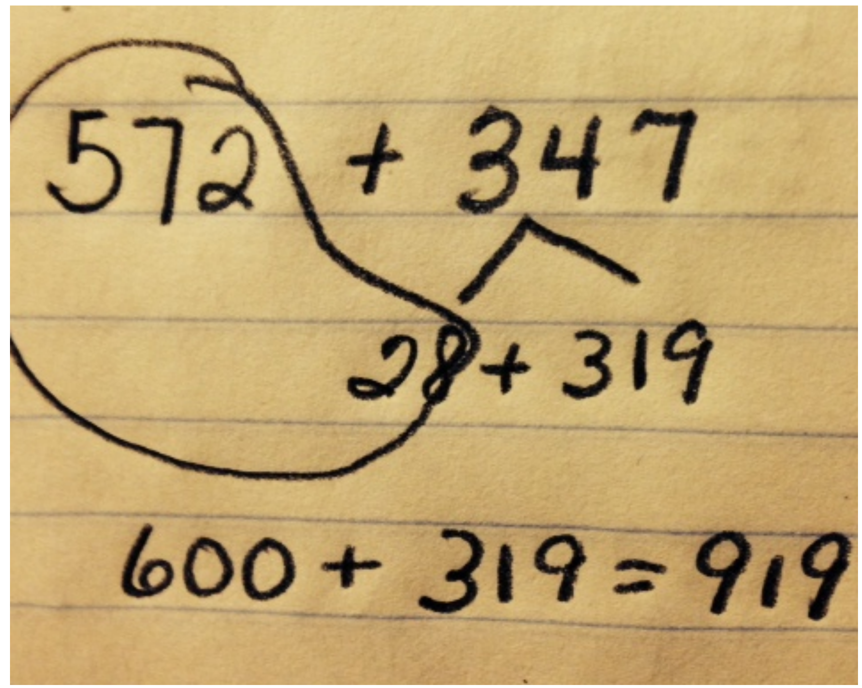
Algorithms

- Partial Sums
  - Preserves place value
  - Left-to-right orientation
  - Supports number sense



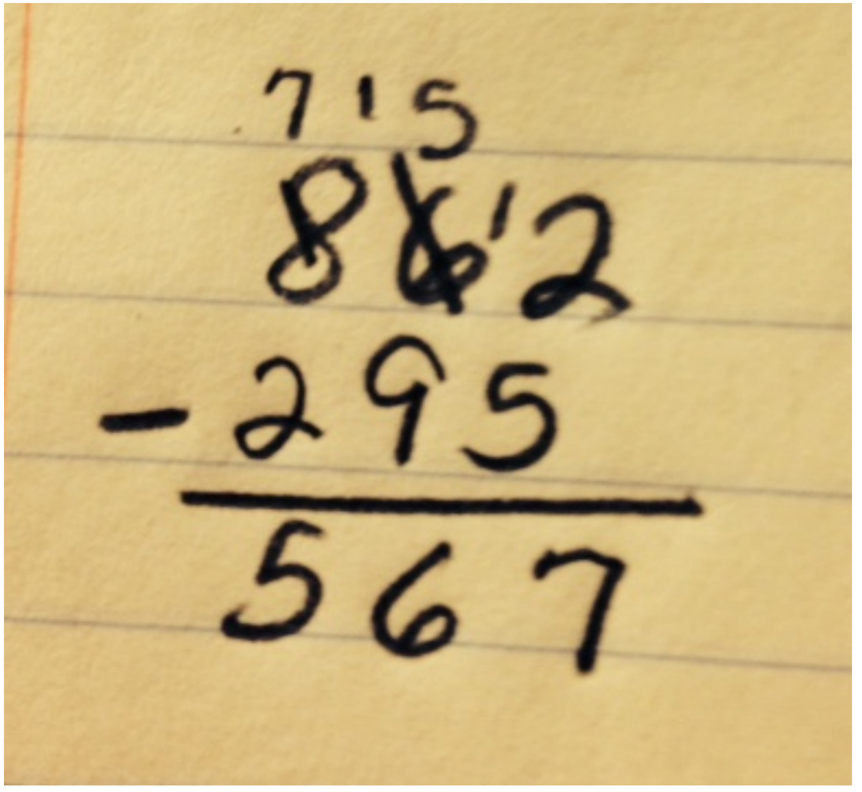
Algorithms

- Round-number strategy
  - Supports number flexibility
  - Uses decomposition
  - Allows for multiple representations



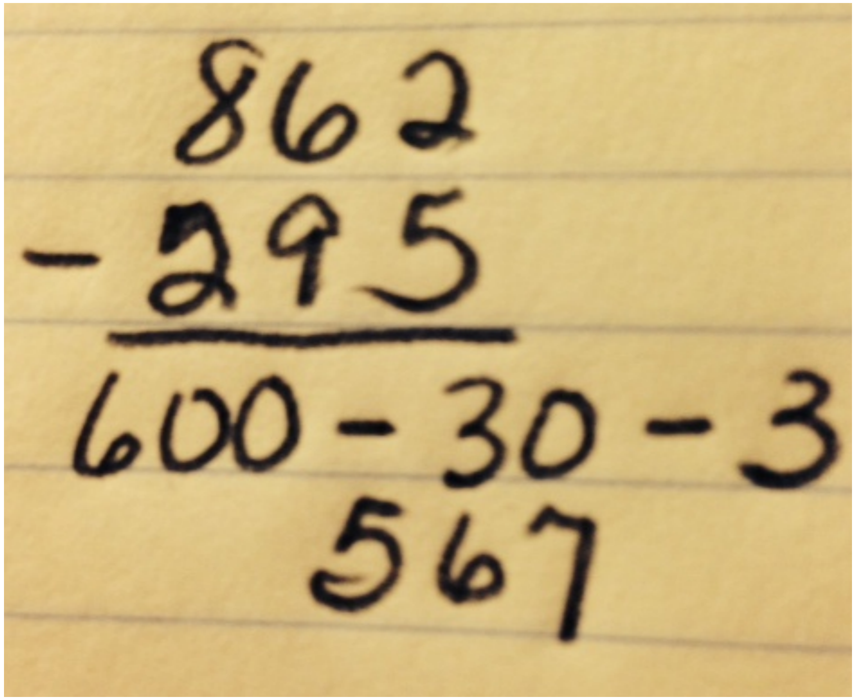
Algorithms

- Traditional
  - Same issues as with traditional algorithm for addition



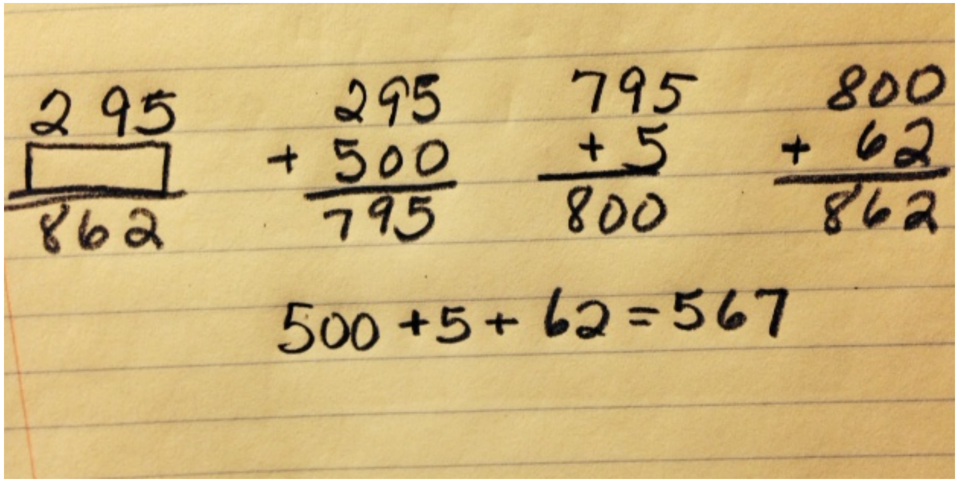
Algorithms

- Partial differences
  - Allows left-to-right direction
  - Supports the idea that you can subtract a smaller number from a larger number



Algorithms

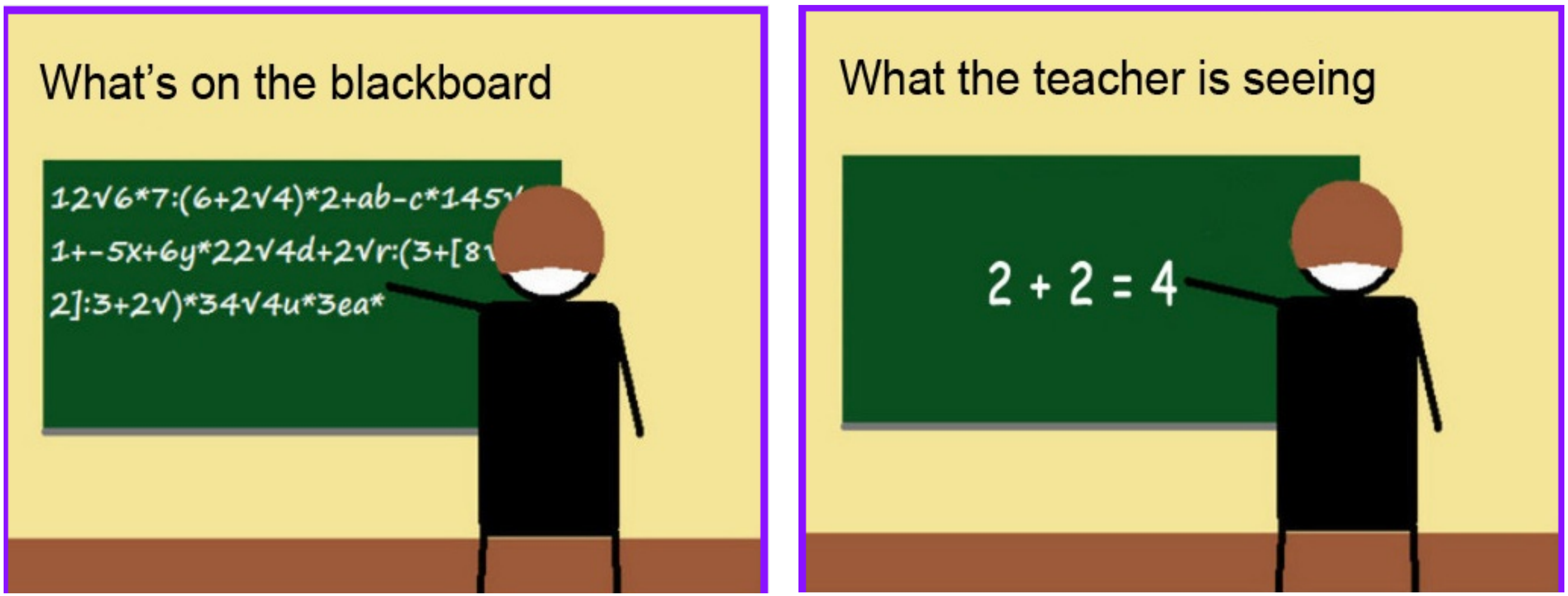
- Missing addend
  - Lessens likelihood of mistake
  - Supports mental computation, mental strategies, and number flexibility

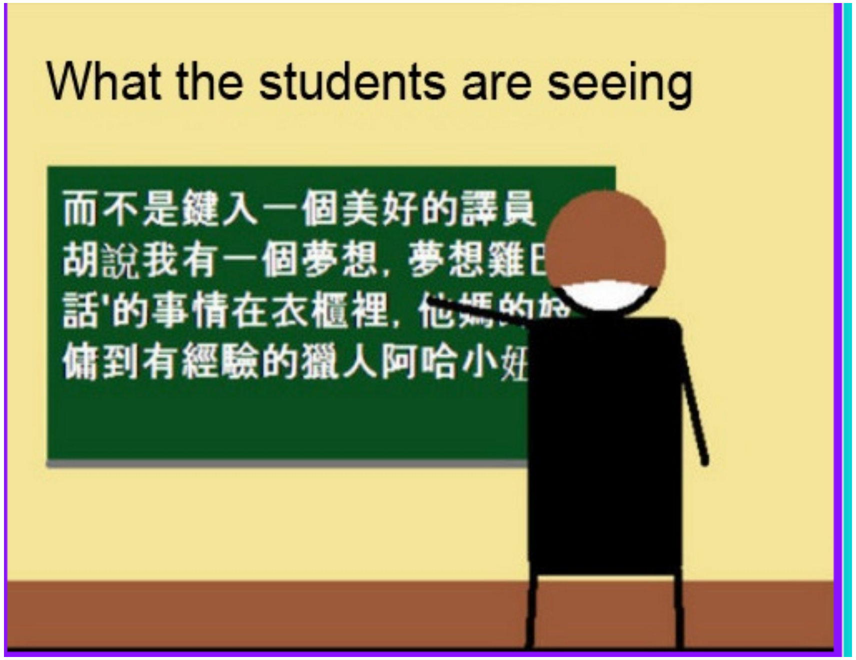


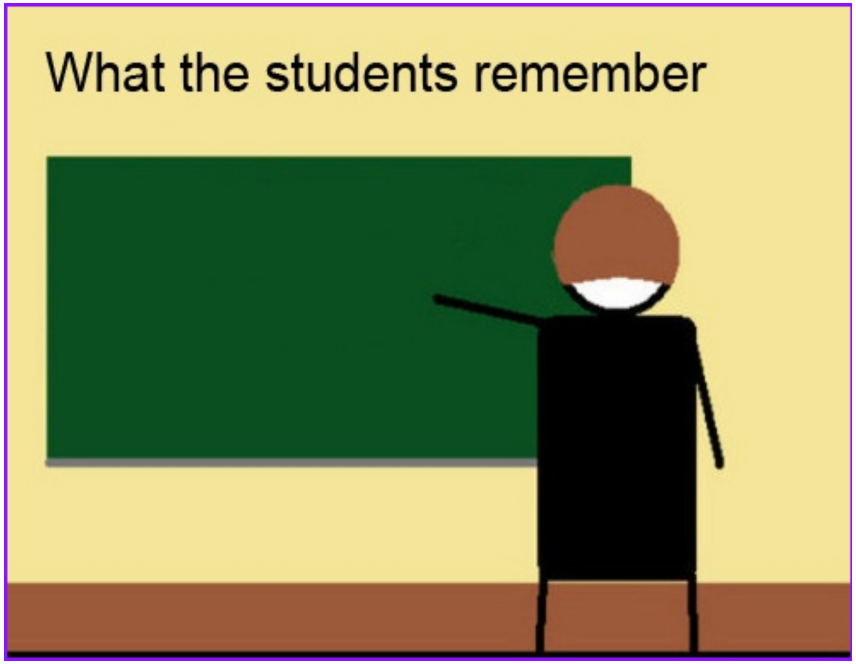
Example based teaching

- Here's how you. . . . . . .
- Now you solve these.

- I do
- We do
- You do


What's on the blackboard
12√6*7:(6+2√4)*2+ab-c*145
1+-5x+6y*22√4d+2√r:(3+[8
2]:3+2√)*34√4u*3ea*
What the teacher is seeing
2 + 2 = 4


What the students are seeing
而不是鍵入一個美好的譯員
胡說我有一個夢想，夢想雞
話'的事情在衣櫃裡，
傭到有經驗的獵人阿哈小


What the students remember

Teaching and Learning

- Telling isn't teaching.

- Told isn't taught.

- Listening isn't learning.

Shifts in teaching and learning

Moving away from . . .	To. . .
Telling/showing how to do something	Building from concept to skill
Teacher-centric instruction	Student-centered instruction
Problem solving intermittently	Problem solving every day
A focus on only the answer	A focus on justifying and explaining
Showing the steps	Explaining the reasoning
Problems that require only fast calculations	Problems that require thinking

What is explicit instruction?

- It is NOT direct instruction.
  - Direct instruction is the teacher showing students how to do something or giving factual information.

What is explicit instruction?

- Focusing students attention on particular structures or ideas
  - Asking questions so that students ‘see’ the mathematics
  - Providing tasks that allow students to explore the topic

What does it look like?

- Teacher introduces a problem that links to previous learning.
- Students work in pairs or small groups to solve.
- Students share their thinking with the class, critiqued by others and teacher.
- Teacher scaffolds tasks based on misconceptions that are evident in thinking.

Explicit Instruction

- Try to elicit the information from students (see Never Say Anything a Kid Can Say)

Patterns: Times 2 Problem
Grade 4

Sophie wrote the following equations:

$$14 = 7 \times 2$$
$$12 = 6 \times 2$$
$$10 = 5 \times 2$$
$$8 = 4 \times 2$$

She said, “I notice something.” What do you think Sophie noticed?

Times 2 Problem

Generalizations:

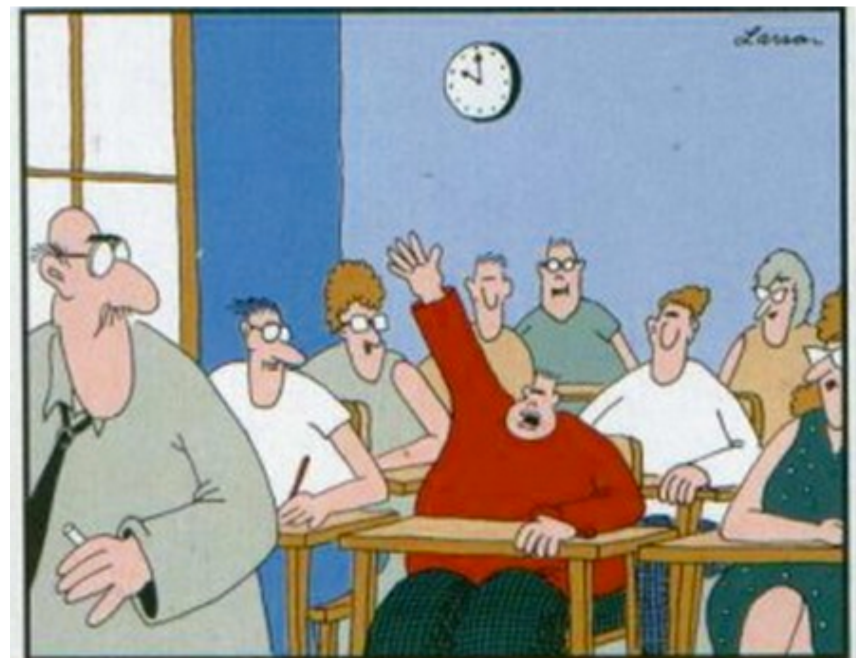
- Multiplying by 2 results in an even product.
- When one factor (2) stays the same, and the other decreases by 1, the product decreases by 2.
- Multiplying by 2 gives a product equal to the sum if you add a number to itself.

Explicit instruction

- Introduce vocabulary (factors, product)
- Relationships when 2 is a factor
- Extension to other factors

Food for Thought

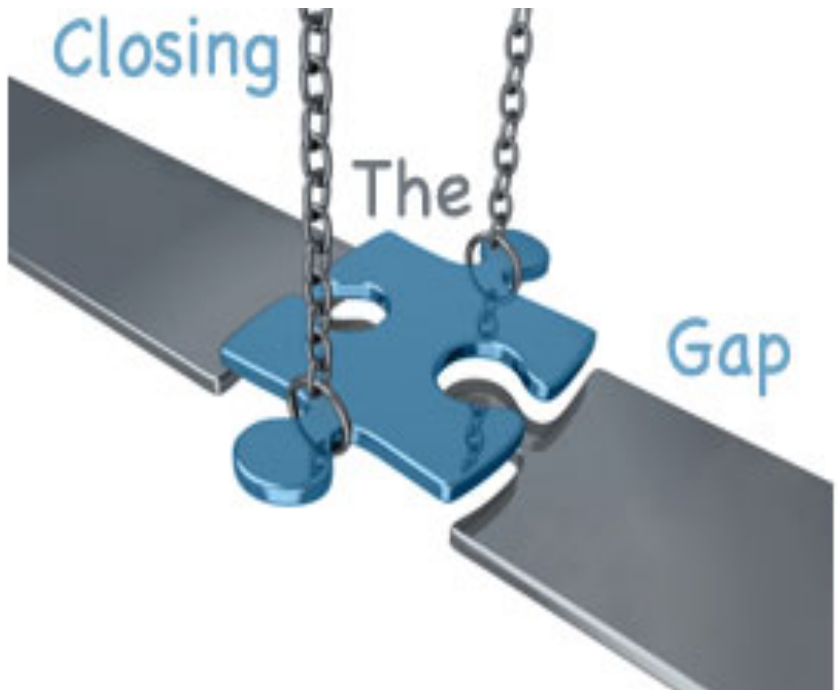
- Critical thinking questions should be asked in every class, every day
- Consistency helps students understand the expectations and move toward higher proficiency



Can I be excused? My brain is full.

Closing the Gap

- Changing the way tasks are posed
- Creating high expectations and accountability



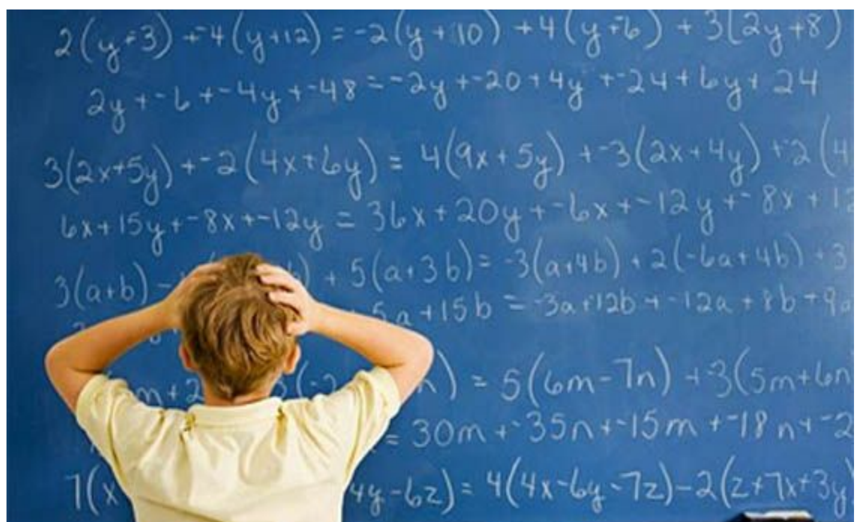
Questioning Techniques

- Factual questions comprise the majority of questions asked in a mathematics class
  - More than 145 questions in 48 minute class period
  - Less than 2 seconds for response

Dougherty & Foegen, 2010

Traditional Tasks

$$\begin{array}{r} 458 \\ +\ 397 \\ \hline \end{array}$$



Change the Task

Reversibility question:

- Find 2 three-digit numbers whose sum is 855
- Find another pair
- Find 3 more pairs

Change the Task

Generalization question:

- What is the maximum number of digits you can get in the sum when you add 2 three-digit numbers? Why?
- What is the minimum number of digits you can get in the sum when you add 2 three-digit numbers? Why?

Change the Task

Flexibility question:

- Add 458 and 397 in two different ways.
- How are the ways you added them alike?
- How are they different?

Change the Task

Flexibility question

Add:

458 + 397 = ?

463 + 397 = ?

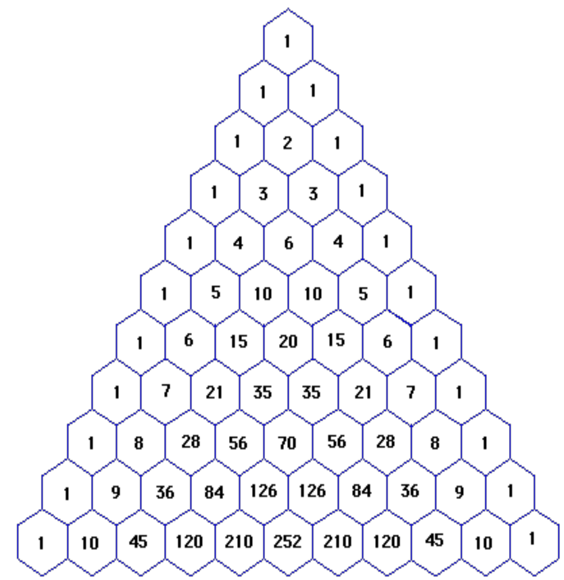
463 + 407 = ?

Questions to Promote Problem Solving and Generalizations

- Reversibility questions:
  - Promote the ability to think in different ways
  - Give the answer, students create the problem

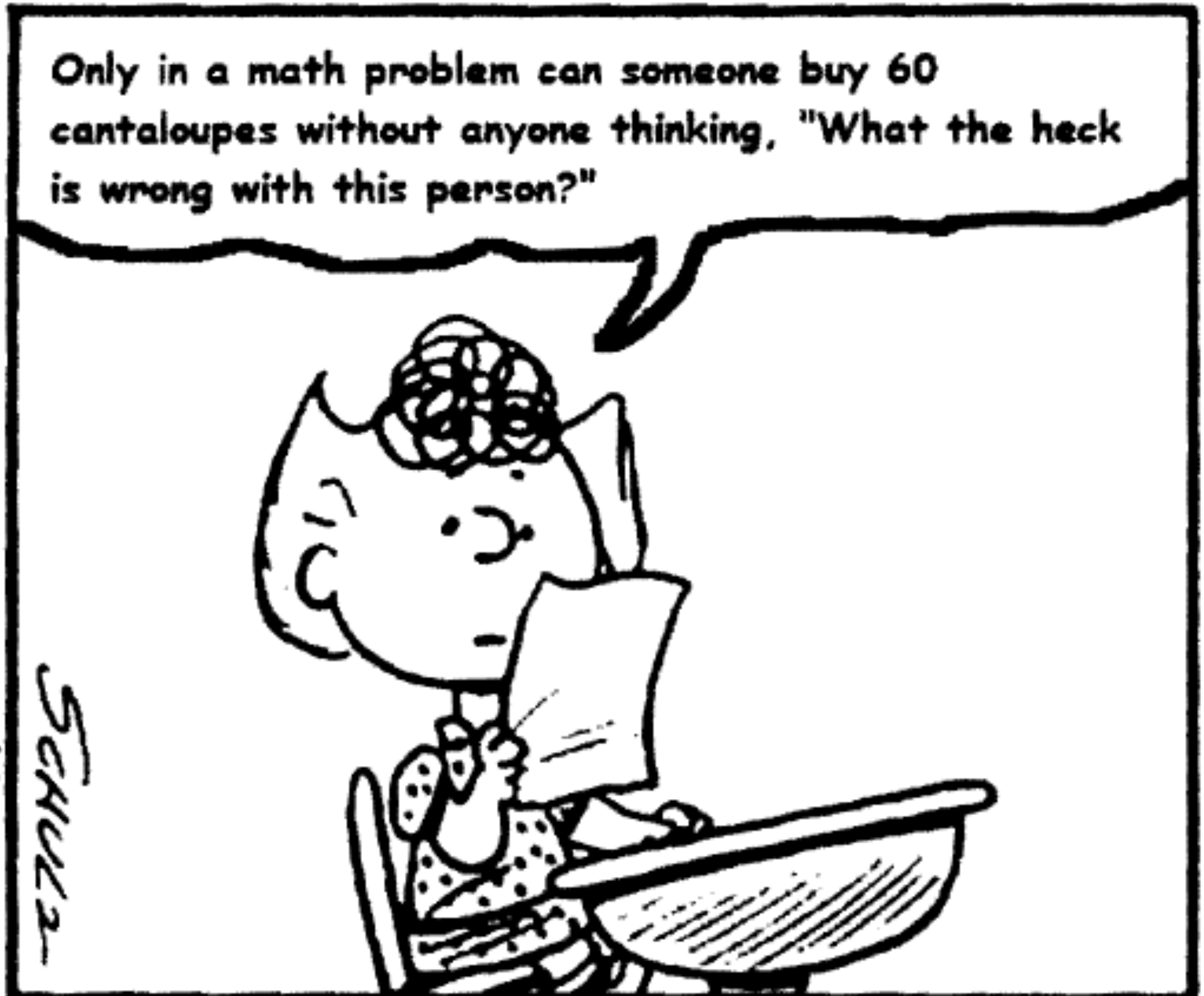
Questions to Promote Problem Solving and Generalizations

- Generalization questions:
  - Ask students to find and describe patterns
  - What patterns do you notice?



Questions to Promote Problem Solving and Generalizations

- Flexibility questions:
  - Ask students to solve a problem in multiple ways OR to use what they know about one problem to solve another one
  - Solve the problem in another way
  - How are these solutions alike? How are they different?


Only in a math problem can someone buy 60 cantaloupes without anyone thinking, "What the heck is wrong with this person?"
SCHULZ
©KARIMI

The Myth of Keywords

- Keywords do not—
  - Develop of sense making or support making meaning
  - Build structures for more advanced learning
  - Appear in many problems

- Students use key words inappropriately

- Multi-step problems are impossible to solve with key words

Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Danger: Key Words Ahead

Mark has 33 packages of pencils. There are 6 pencils in each package. How many pencils does he have <u>in all</u>?

39 because it says in all.

The Infamous Shepherd Problem

:

There are 25 sheep and 5 dogs in a flock. How old is the shepherd?

How do you think students solved the problem?

Caldwell, Kobett & Karp (forthcoming) Addition and Subtraction in Practice. NCTM.

Results from 214 Students

	Added the numbers	Subtracted the numbers	Multiplied the numbers	Created a ratio	Other Incorrect procedure	Suggested no solution is possible
Third grade (n = 58)	76%	8%	0%	0%	14%	2 %
Sixth grade (n = 71)	48%	9%	21%	8%	6%	8 %
Seventh grade (n = 85)	48%	2%	17%	14%	9%	10 %

Caldwell, Kobett & Karp (forthcoming) *Putting essential understandings into practice in grades preK–2: Addition and subtraction.* A volume in the series edited by B. J. Dougherty, National Council of Teachers of Mathematics..

Really?

Caldwell, Kobett & Karp (forthcoming) *Putting essential understandings into practice in grades preK–2: Addition and subtraction.* A volume in the series edited by B. J. Dougherty, National Council of Teachers of Mathematics..

Not All Problem Solving is Created Equal

- Application/routine problems: solved by an algorithm
- Non-routine problems: a creative solution approach is needed

Do you think differently when you solve these problems?

Brett wanted to put carpet on his bedroom floor. His room measures 10’ X 15’. How many square feet of carpet does he need?

You have 8 coins and a balance scale. The coins look alike, but one is counterfeit and lighter than the other 7. Find the counterfeit coin using 2 weighings on the balance scale.

Do you think differently when you solve these problems?

Word/Application

Brett wanted to put carpet on his bedroom floor. His room measures 10’ X 15’. How many square feet of carpet does he need?

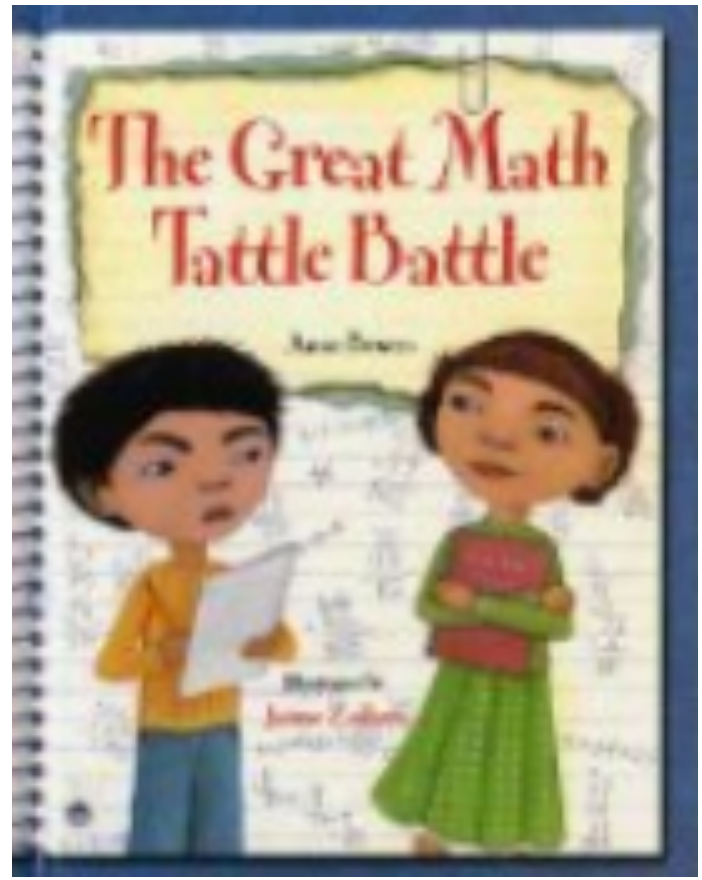
Non-routine

You have 8 coins and a balance scale. The coins look alike, but one is counterfeit and lighter than the other 7. Find the counterfeit coin using 2 weighings on the balance scale.

Use Literature

Great Math Tattle Battle

by Anne Bowen



Harley Harrison

$$\begin{array}{r} 35 \\ +64 \\ \hline 98 \end{array} \qquad \begin{array}{r} 18 \\ +18 \\ \hline 216 \end{array} \qquad \begin{array}{r} 50 \\ -31 \\ \hline 21 \end{array}$$

$$\begin{array}{r} 67 \\ +67 \\ \hline 134 \end{array} \qquad \begin{array}{r} 44 \\ -28 \\ \hline 24 \end{array} \qquad \begin{array}{r} 99 \\ +99 \\ \hline 188 \end{array}$$

Harley Harrison

$$\begin{array}{r} 35 \\ +64 \\ \hline \cancel{9}\cancel{8} \end{array} \qquad \begin{array}{r} 35 \\ +64 \\ \hline 99 \end{array}$$

he added this part wrong

$$\begin{array}{r} 1 \\ 18 \\ +18 \\ \hline \cancel{216} \end{array} \qquad \begin{array}{r} 18 \\ +18 \\ \hline 36 \end{array}$$

he forgot to regroup. The one from the 16 should go at the top.

$$\begin{array}{r} 4\ \ 10 \\ \cancel{5}\cancel{0} \\ -31 \\ \hline \cancel{21} \end{array} \qquad \begin{array}{r} 4 \\ \cancel{5}0 \\ -31 \\ \hline 19 \end{array}$$

he switched those around

he forgot to regroup.

$$\begin{array}{r} 67 \\ +67 \\ \hline 134 \end{array} \qquad \begin{array}{r} 67 \\ +67 \\ \hline 134 \end{array}$$

he got the write answer, but forgot to regroup

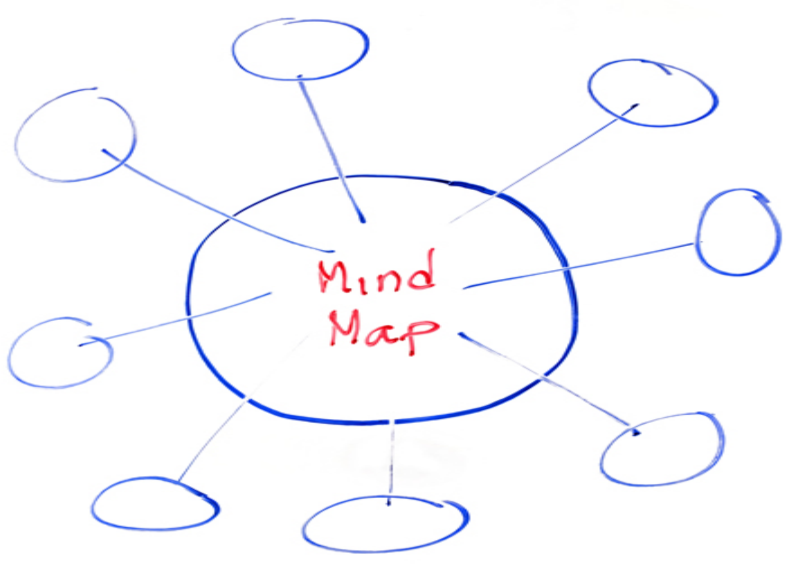
$$\begin{array}{r} 3 \\ \cancel{4}14 \\ -28 \\ \hline \cancel{24} \end{array} \qquad \begin{array}{r} 3 \\ \cancel{4}14 \\ -28 \\ \hline 16 \end{array}$$

he forgot to regroup

$$\begin{array}{r} 1 \\ 99 \\ +99 \\ \hline \cancel{188} \end{array} \qquad \begin{array}{r} 99 \\ +99 \\ \hline 198 \end{array}$$

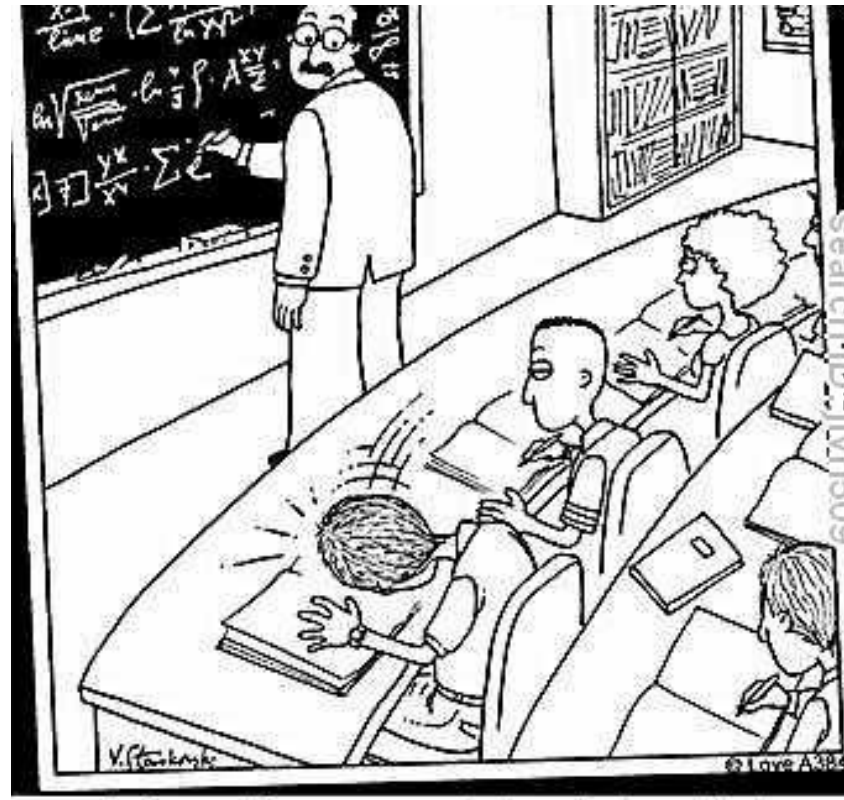
he fogot to regroup

What are general classroom strategies?



Routines

- Response routines
  - Think-pair-share
  - Pair think/response
  - Random calling
  - Unison response
  - Display student work
    - Whiteboard or other display of answers

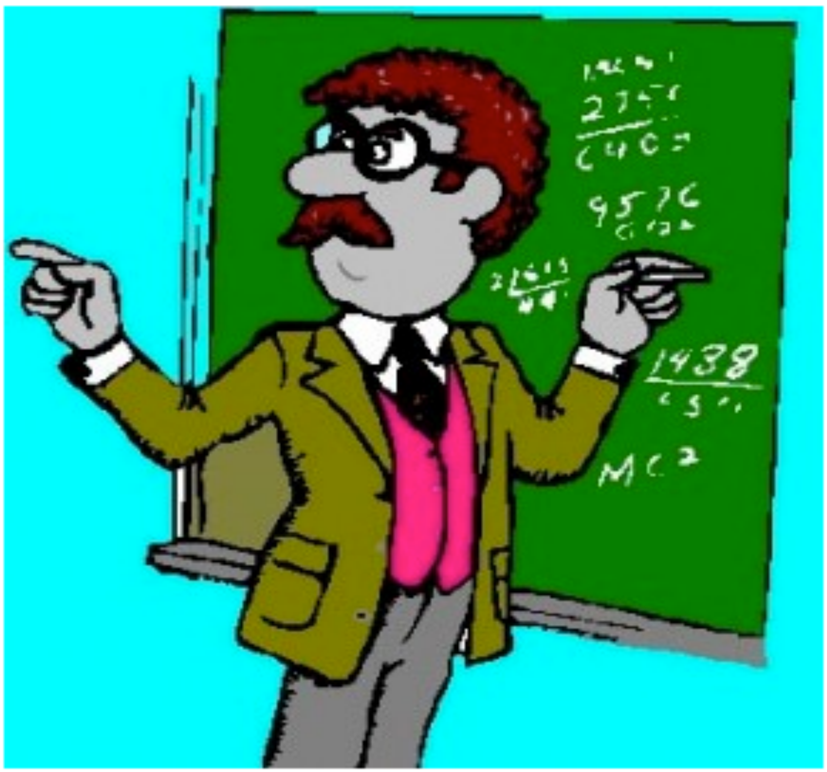


Professor Herman paused when he heard that unmistakable thud -- another brain had imploded.

Routines

- Response routines
  - Wait time
  - Give students time to think (3 – 5 seconds?)

Student responses



- If discussing, honor student responses by writing them
- Focuses students' attention

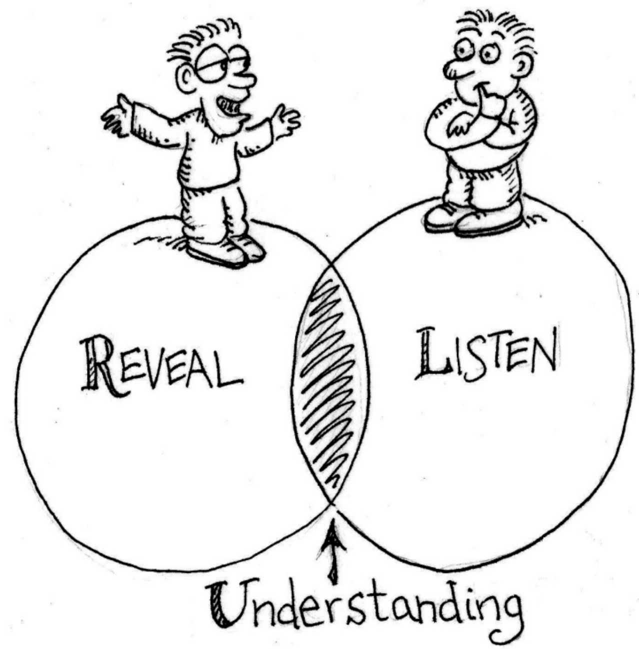
Instructional Routines

- Name them
- Give an advance organizer for the day
- Practice them

"I find if you put that slash through the equal sign, the number of possible answers vastly increases."

Feedback

- Self monitoring
- Language-based (hearing one's thinking, metacognition)



Curriculum Resources

- Two recent studies revealed that teachers providing Tier 2 mathematics instruction to K–12 students largely used worksheets (Foegen & Dougherty, 2010; Swanson, Solis, Ciullo & McKenna, 2012)
- One-size-fits-all computer programs

Curriculum Resources - Materials

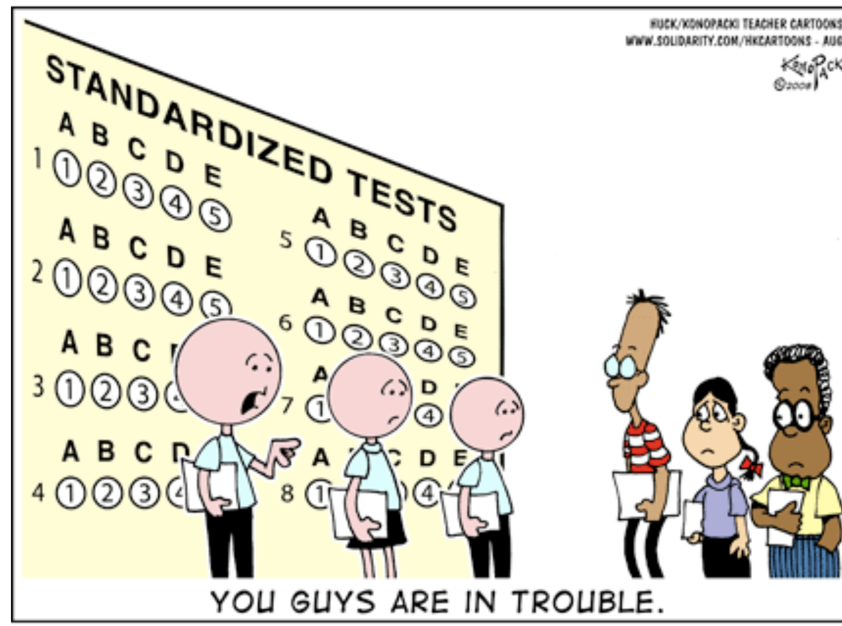
- Effective materials (not just research-based)
- Manipulatives
- Different from Tier 1 classes

THE FAR SIDE / GARY LARSON

Hell's library

Assessment

- If assessments only measure skill, it is difficult to determine what a student knows
- If you cannot determine what a student knows, it's difficult to plan an instructional sequence

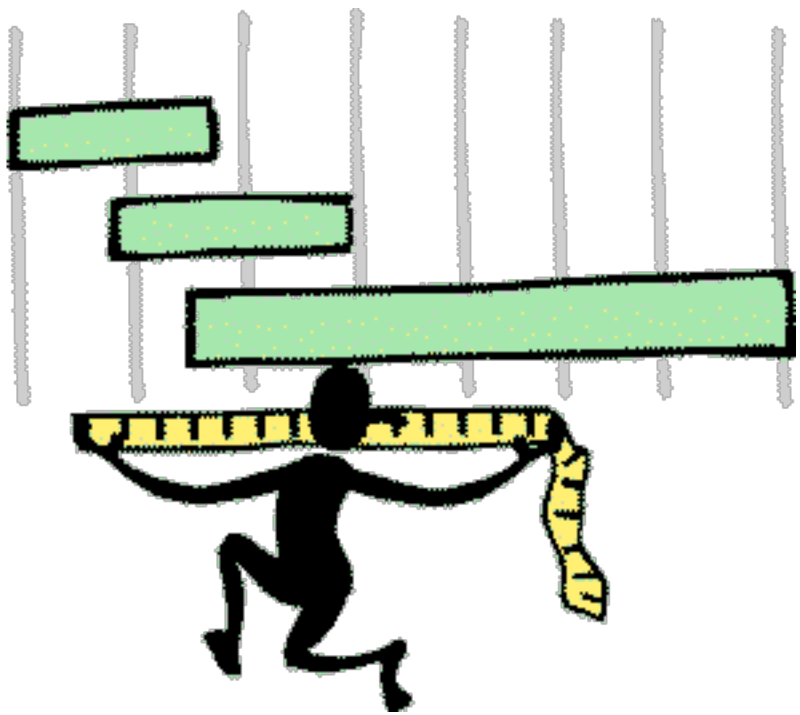


Data Resources

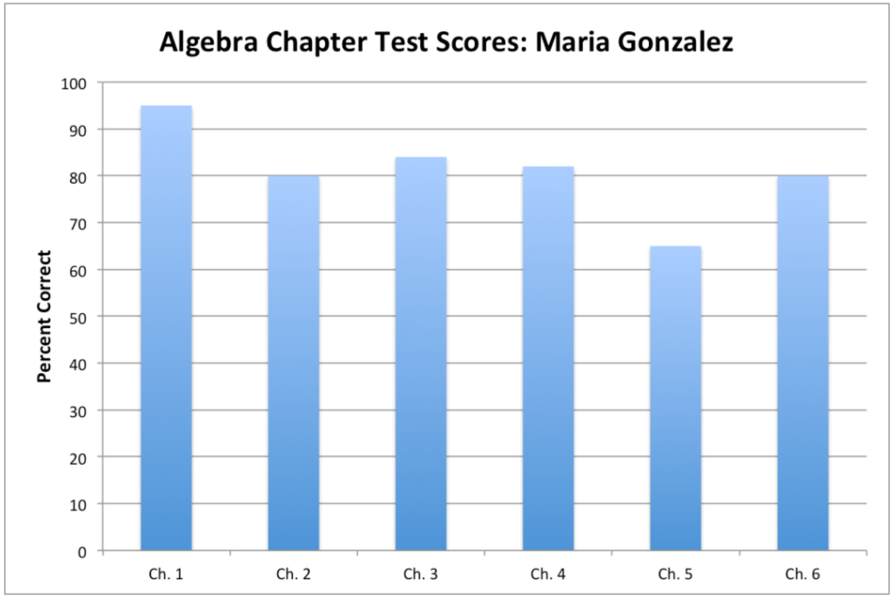
- Universal screener
- Progress monitoring

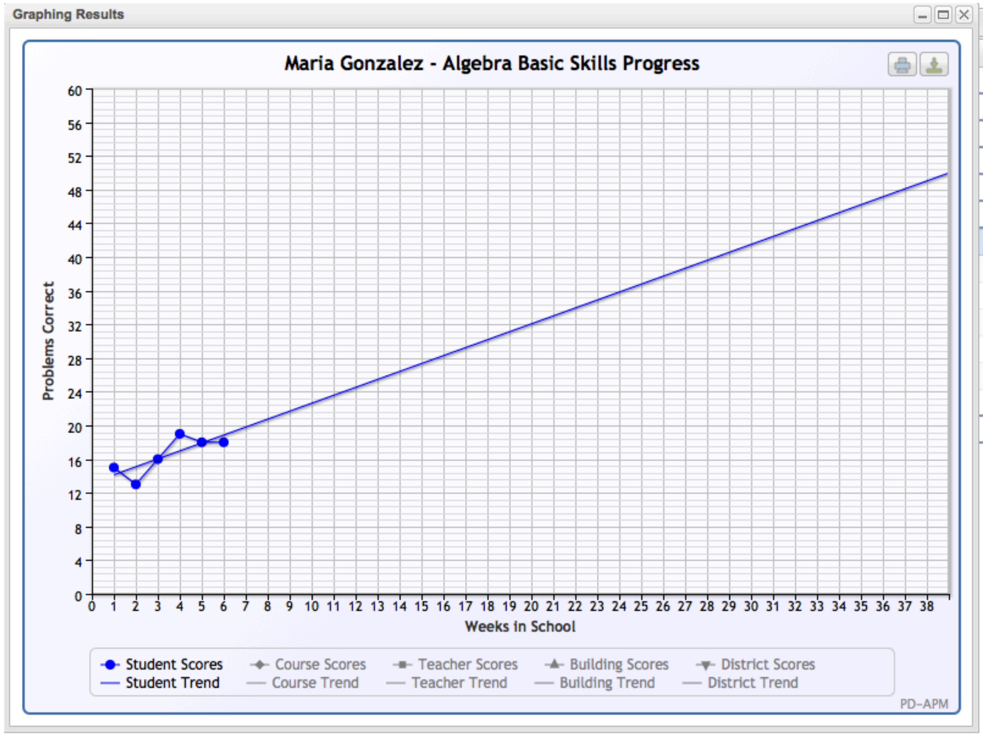
Progress Monitoring

How is progress monitoring different from classroom instruction?

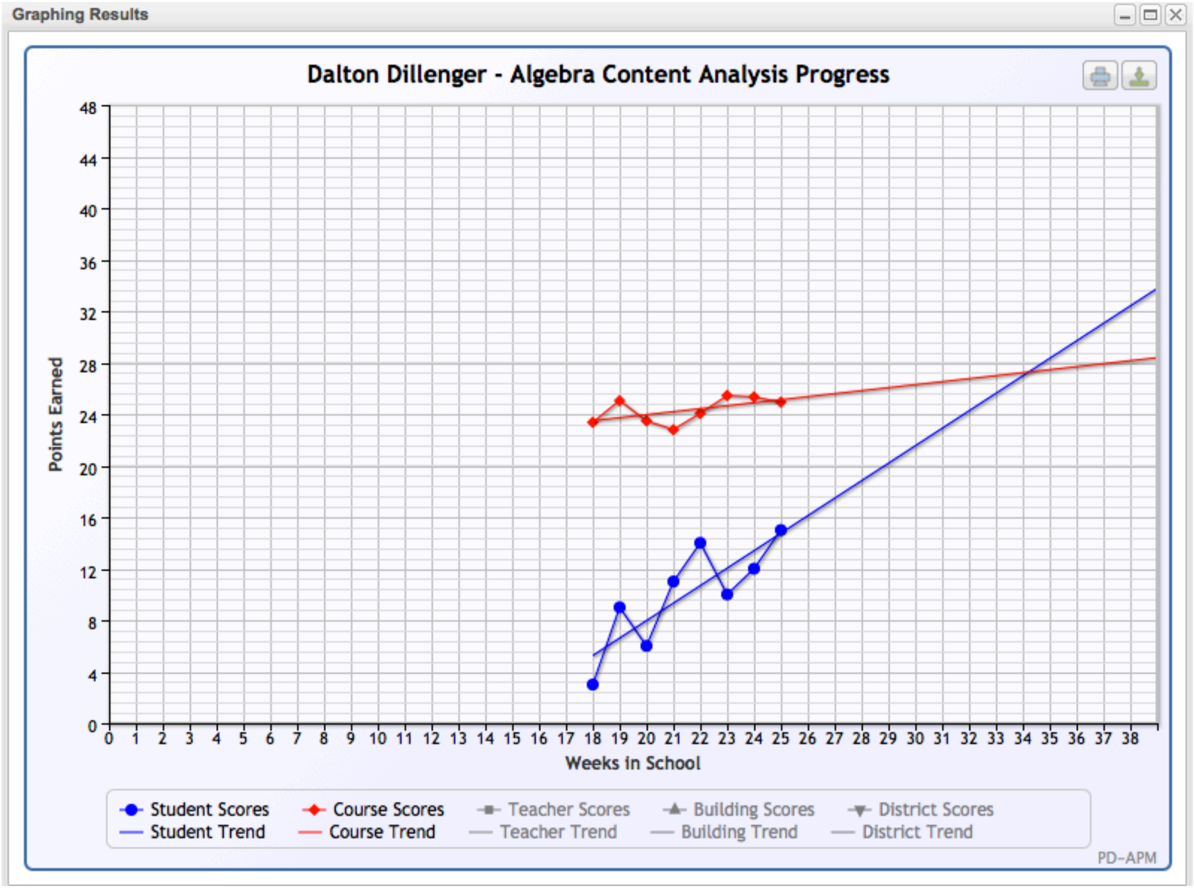


Progress Monitoring Unique Features





Graph for Progress Monitoring



Data Resources

- How will you determine who is placed in Tier 2 instruction?
- How will you determine who leaves Tier 2 instruction?

Progress Monitoring Assessment

- Procedural measures: skills, algorithms
- Typically multiple-choice format

Interpreting student errors

Write the fraction that is represented by the figure:

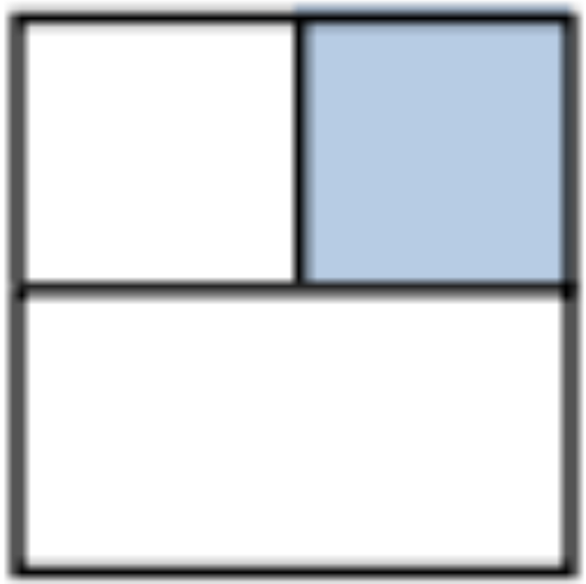


Figure 1.

Common Response

One-third

Why do you think students would say that?

What could we do to help them develop a better understanding?

Interpreting student errors

A student does the following multiplication problem:

$$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$$

Look at the statement below:

$\frac{10}{12}$ is twice as large as $\frac{5}{6}$.

Decide whether you agree or disagree with the statement.

Agree Disagree

Interpreting student responses

Agree

Why do you think students would say that?

What could we do to help them develop a better understanding?

Teacher Resources

We expect that the very best doctors will treat the most grievously ill patients. It should be no different in education. Great teachers have the skills to help the students who struggle the most.

Larson, M. (2011).. Supporting students' achievement of the Common Core. NCTM Conference presentation. Indianapolis, IN.

Closing Discussion

- Questions?
- Lingering issues?

"I'm afraid I still have more questions than answers."

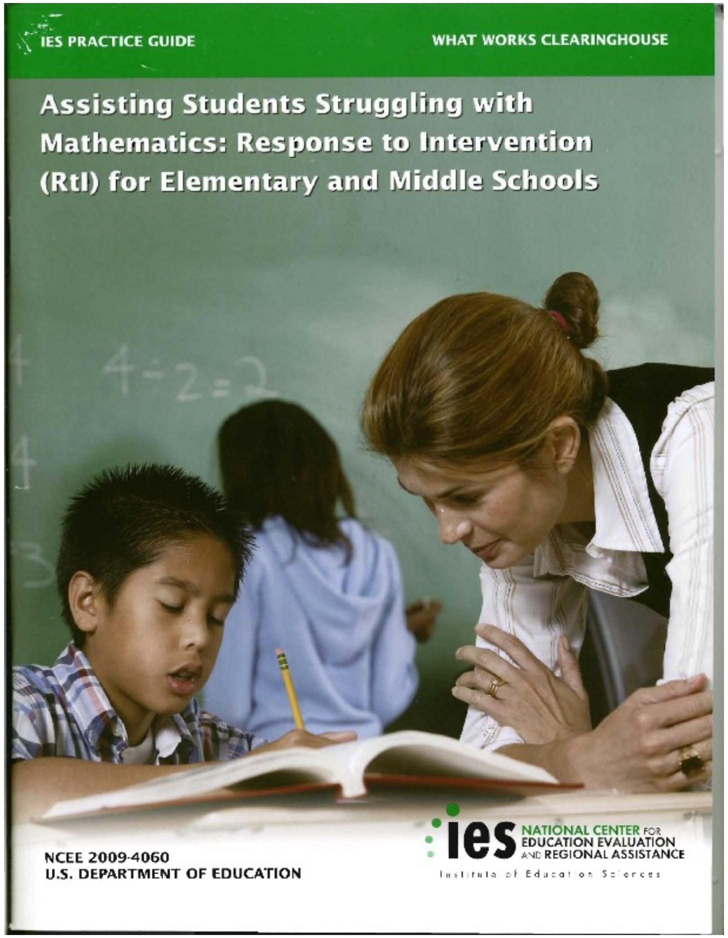
NCTM Resources

- Math of Tomorrow (MOTO Project)
- Pre-conference institute: New Orleans, April 9
- Summer Institutes

Recommendations for Identifying and Supporting Students Struggling in Mathematics

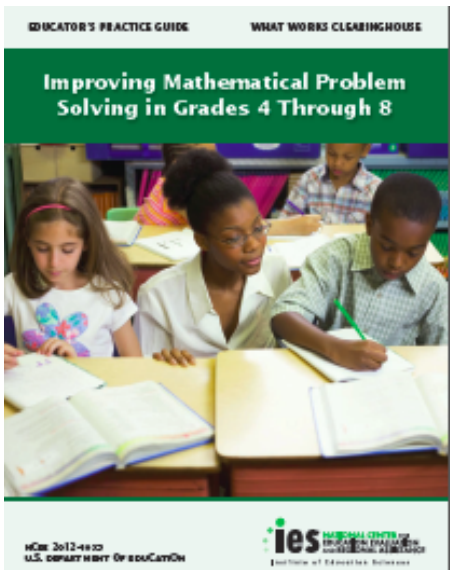
- Recommendations are based on **strong** and **moderate** levels of evidence resulting from comprehensive reviews of current research literature

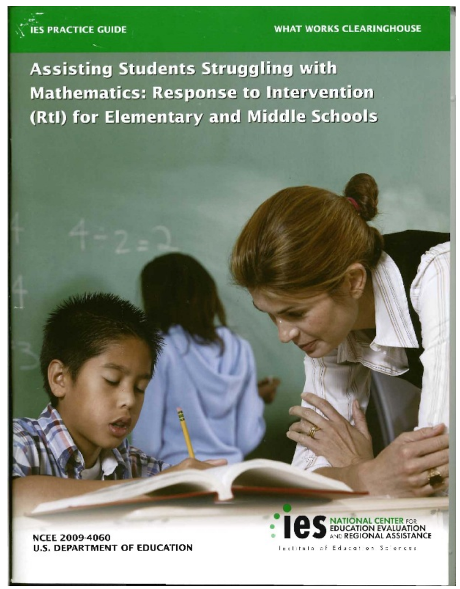
http://ies. ed.gov/ncee/wwc/publications/practiceguides/

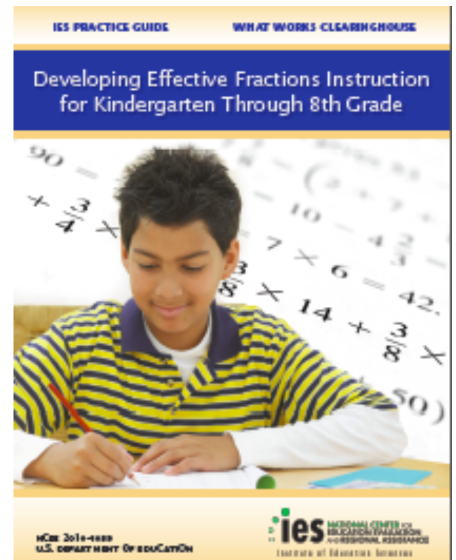


Recommendations for Identifying and Supporting Students Struggling in Mathematics

- Based on **strong** and **moderate** levels of evidence resulting from comprehensive reviews of current research







Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC:* National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies. ed.gov/ncee/wwc/publications/practiceguides/.